The Laboratory for Physical Sciences

Quantum Limit of NEMS I

How to Engineer Continuous Position Detection at the Uncertainty Principle Limit

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from *The Onion*

My Group and Collaborators

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Collaborators…

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Chris Monroe	Univ. of Michigan
Kamil Ekinci	Boston University
Pierre Echternach	JPL / Caltech

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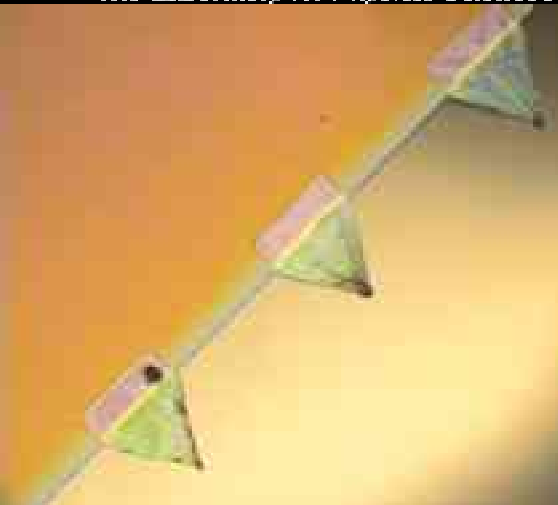
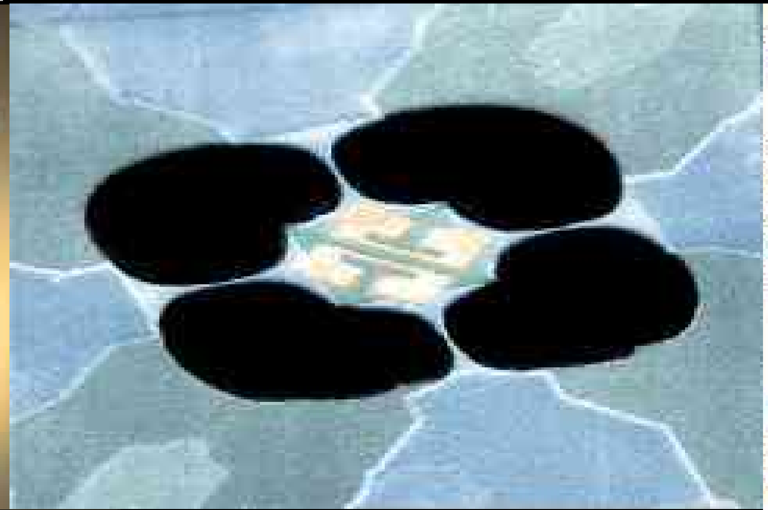
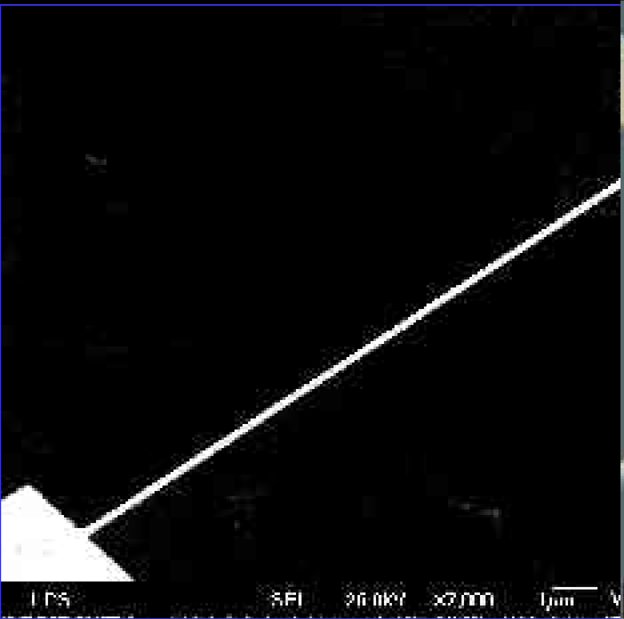
Examples of Nanomechanical Devices



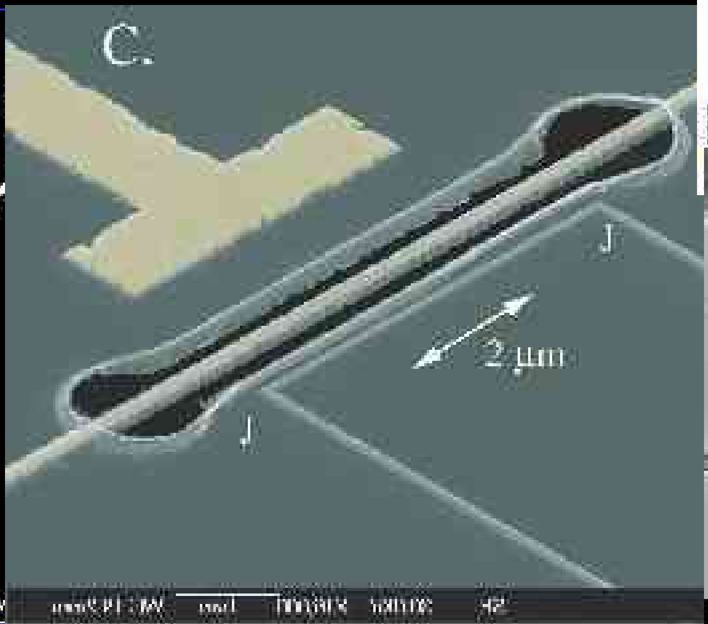
MRFM detectors



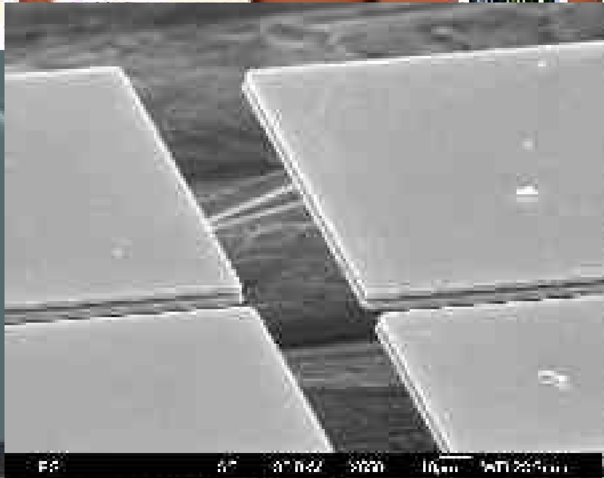
Quantized Thermal Transport



Simple Nanomechanical Resonators



Integrated rf SET



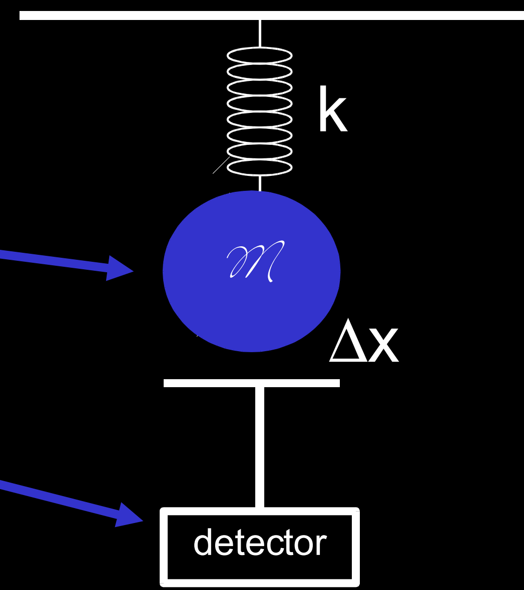
Atomic Traps

Quantum limits of position detection

There are two pieces to this system which have quantum limits:

Massive resonator

Amplifier

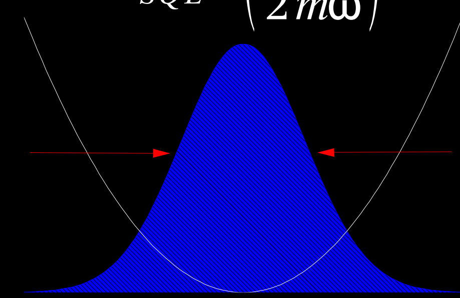


For one instantaneous measurement: *no limit*

For two instantaneous measurements: $\Delta x_{SQL} = \left(\frac{\hbar}{2 m\omega} \right)^{1/2}$

For a continuous measurement: $\Delta x_{QL} = \left(\frac{\hbar}{\ln 3 \cdot m\omega} \right)^{1/2}$

$$\Delta x_{SQL} = \left(\frac{\hbar}{2 m\omega} \right)^{1/2}$$



$$T_N = \frac{h\nu}{k_B \cdot \ln 3} \Rightarrow \Delta x_{QL} = \left(\frac{\hbar}{\ln 3 \cdot m\omega} \right)^{1/2}$$

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How close have other come?

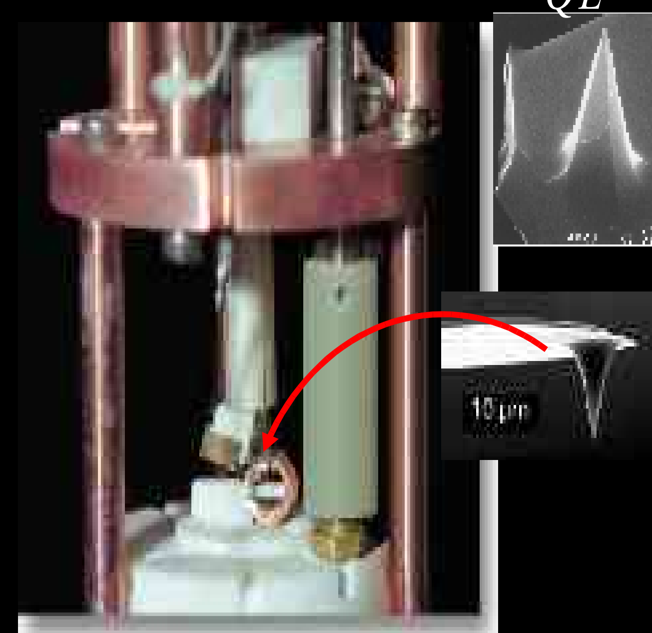
4 km Interferometers - LIGO

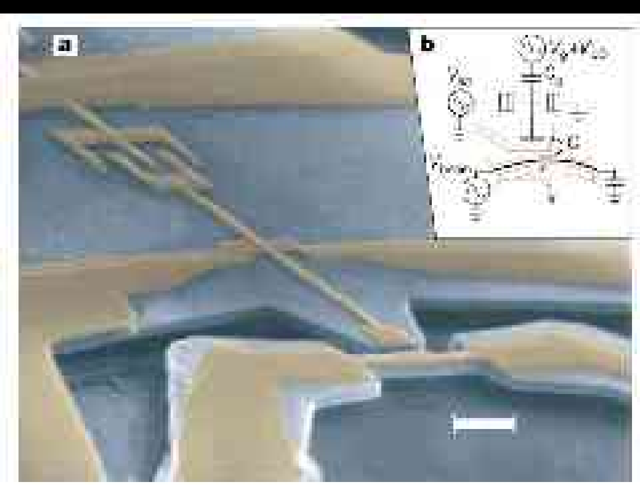
$$\Delta x \approx 100-1000\Delta x_{QL}$$

2 Ton Acoustic Resonators-Auriga $\Delta x \approx 167\Delta x_{QL}$

Single-Spin Microscopes

$$\Delta x \approx 1500\Delta x_{QL}$$





SET+Nanomechanics

$$\Delta x \approx 100\Delta x_{QL}$$

Energy Quantization - Freeze-Out

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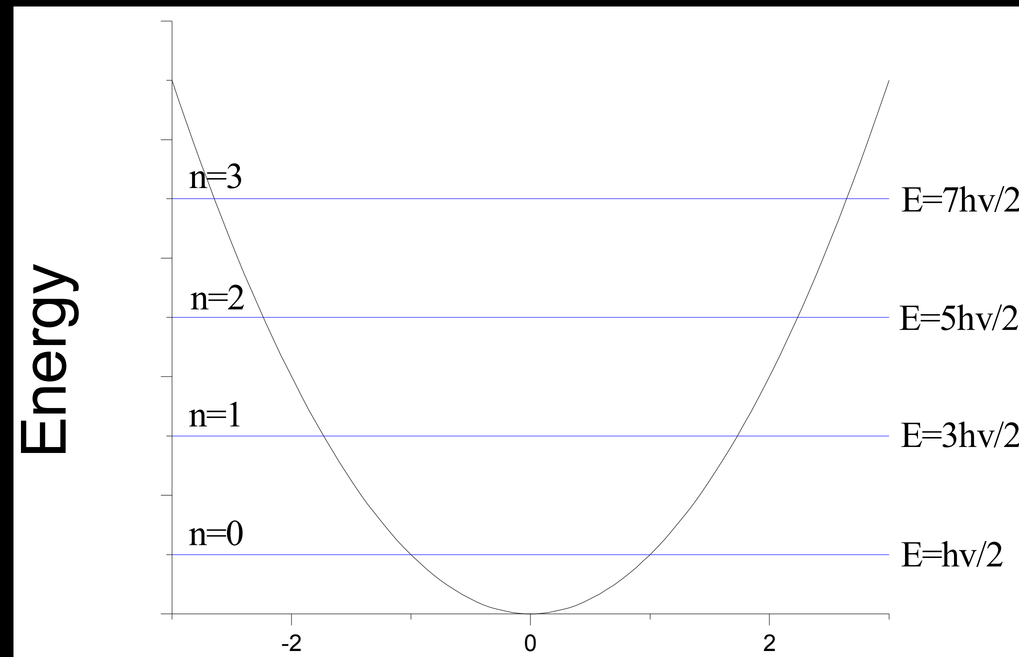
$$\hat{H} = \frac{1}{2} k \hat{x}^2 + \frac{1}{2}\frac{\hat{p}^2}{m}$$

$$\Rightarrow E_n = \hbar\omega\left(n + \frac{1}{2}\right) \qquad n = 0, 1, 2, \ldots.$$

$$\langle E \rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega / k_B T} - 1}$$



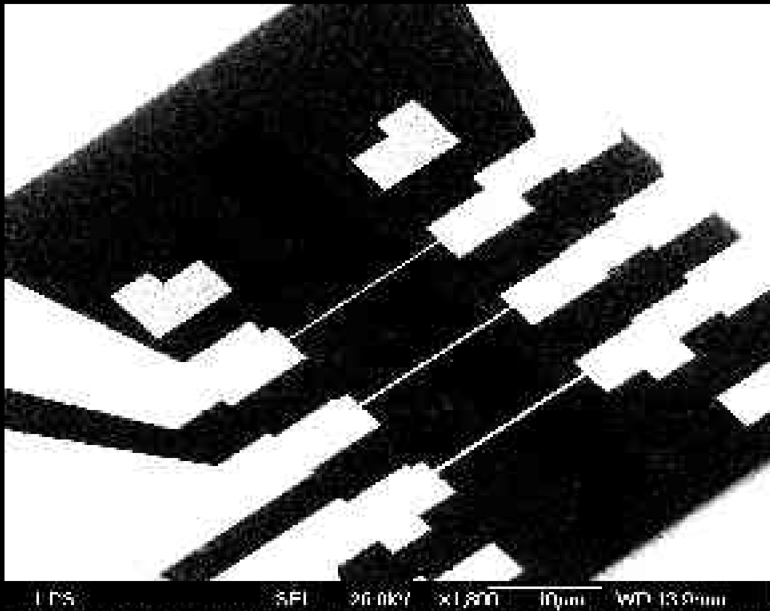
Oscillator occupation number

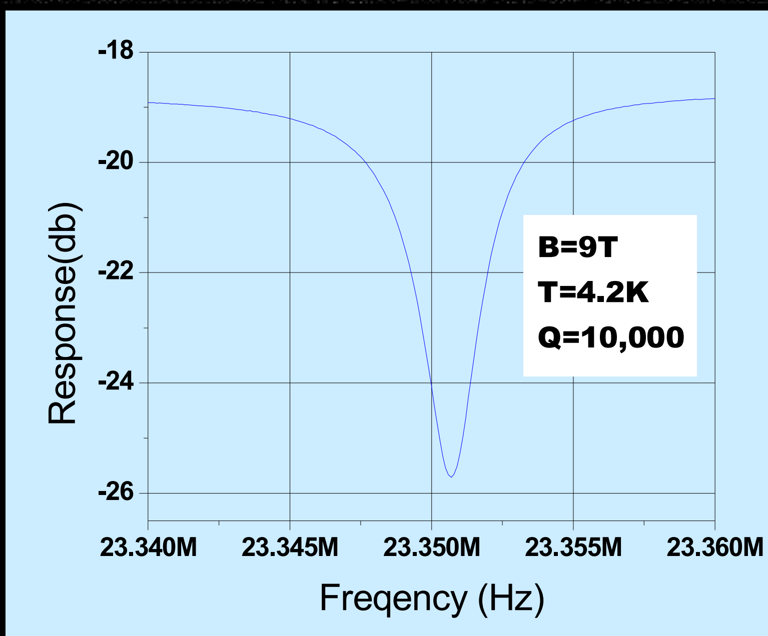
T(mK)	n(1kHz)	n(10MHz)	n(100MHz)	n(1GHz)
1000	$2\ 10^{7}$	2070	207	20
100	$2\ 10^{6}$	207	20.7	1.6
10	$2\ 10^{5}$	20.8	1.6	0.008
1	$2\ 10^{4}$	1.6	0.008	10^{-21}
0.50	$1\ 10^{4}$	0.6		

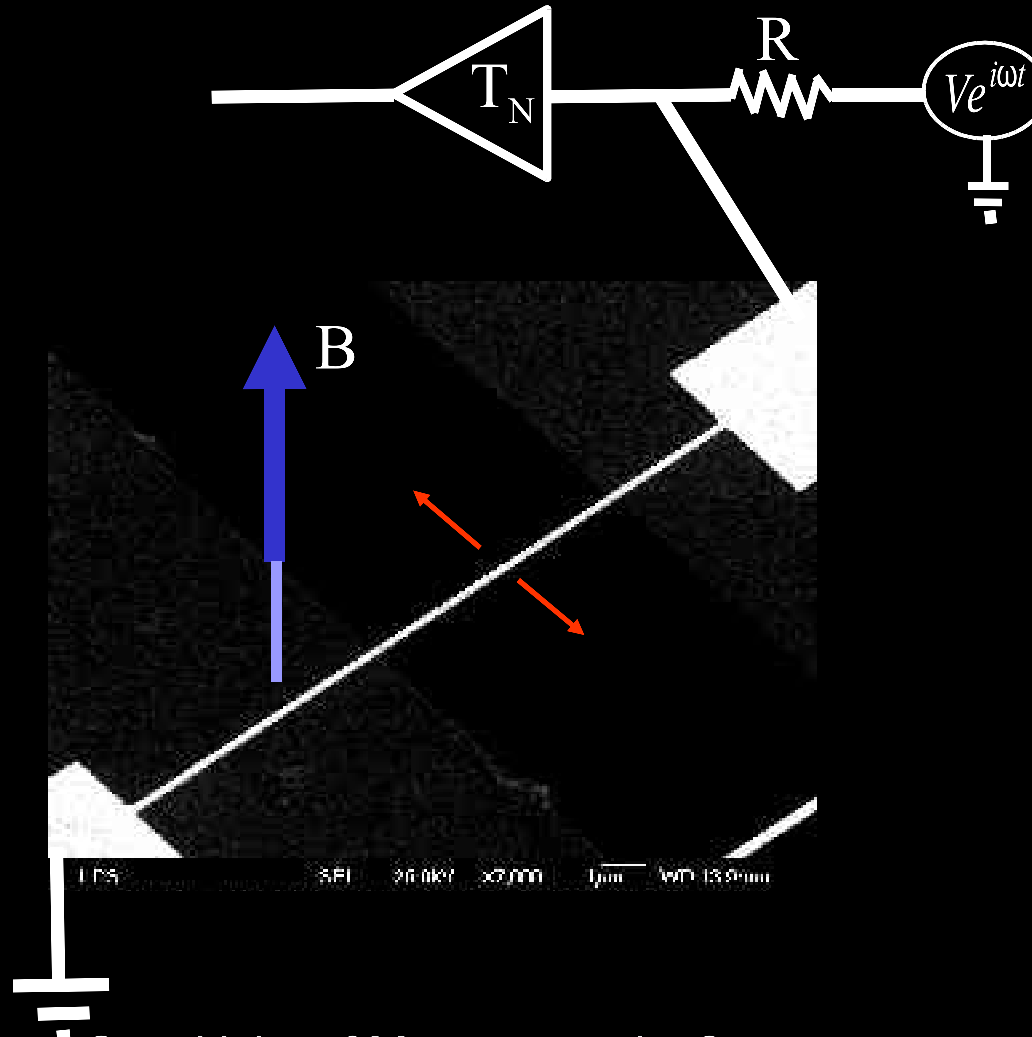
Optimization of position detection….

noise temperatures and all that….

Magnetomotive Readout of Nanomechanics







Sensitivity of Magnetomotive?

Fabrication by LaHaye and Hutchinson

Magnetomotive Detection

Force on resonator by driving oscillating current

(current I, resonator length l, quality factor Q, spring constant k):

$$F = BIle^{i\omega t}$$

Response on resonance:

$$X = Q\frac{F}{k}e^{i\omega t} = \frac{BIlQ}{k}e^{i\omega t}$$

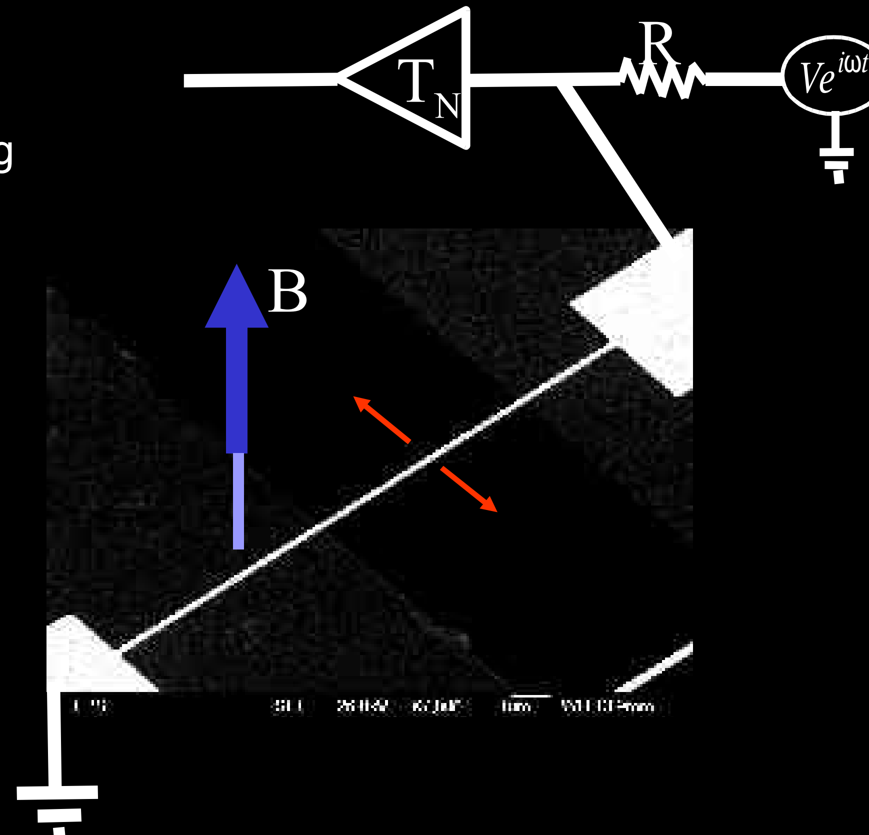
Motion will generate voltage in magnetic field:

$$V = BL\omega \qquad Xe^{i\omega t} = \frac{Q(Bl)^2\omega}{k} Ie^{i\omega t}$$

Effective magnetomotive impedance:

$$Z = \frac{Q(Bl)^2\omega}{k}$$

typical impedance ~0.1Ω to 10,000Ω



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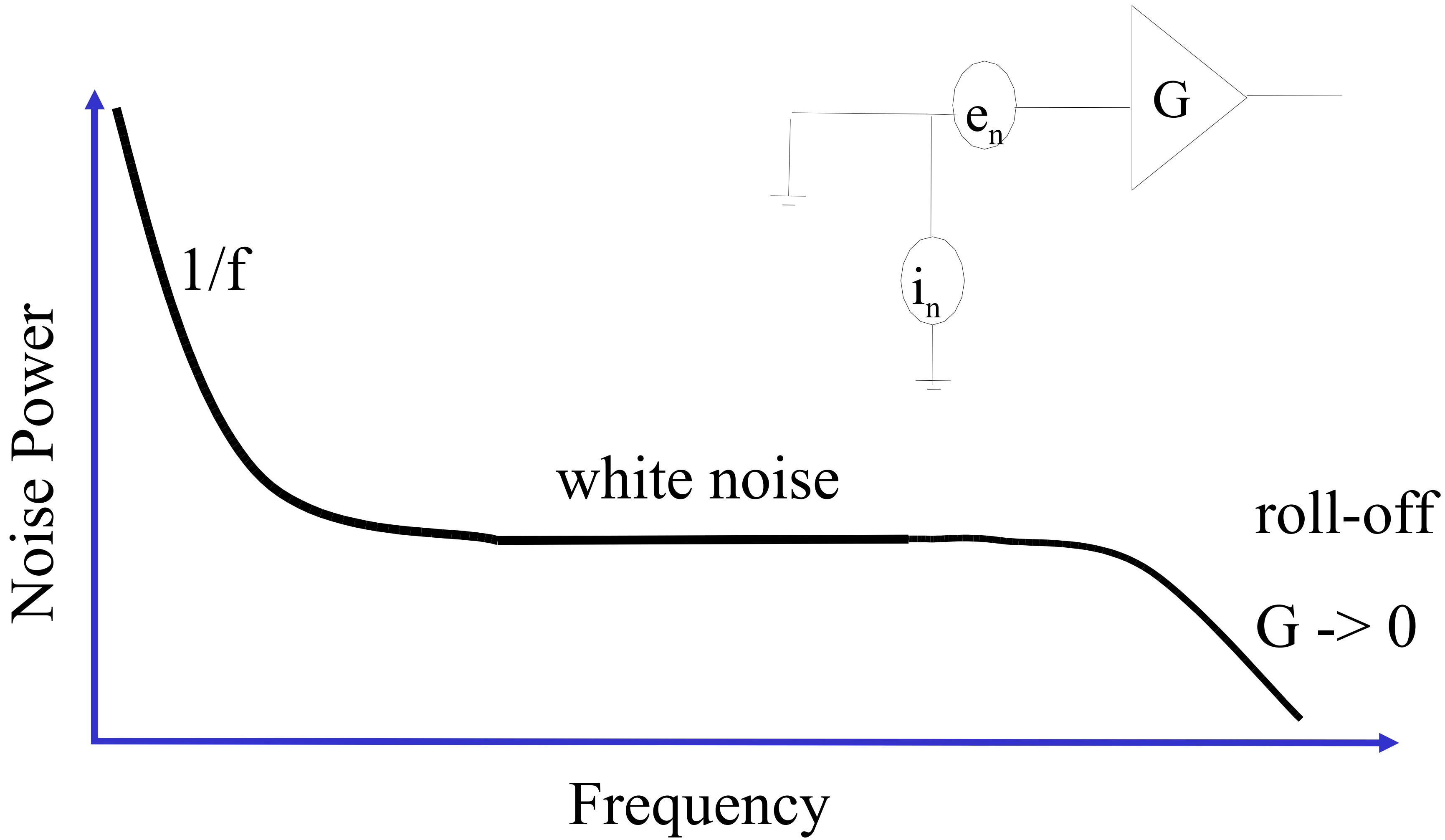
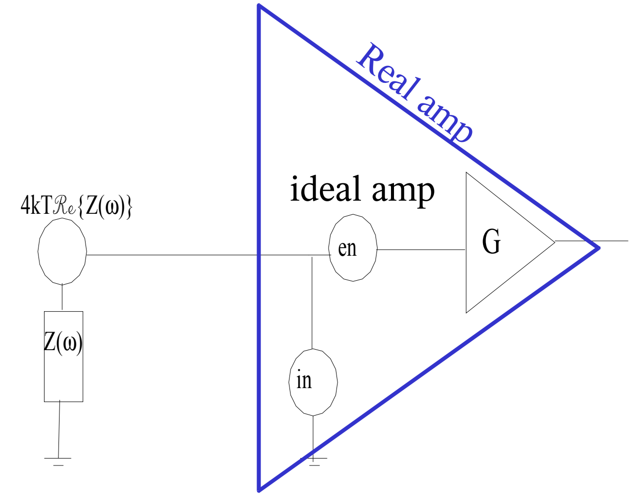
Noise in Amplifiers



Practical Limits of Measurement (classical)



Total voltage noise appearing at the input:

$$V_T^2 = e_n^2 + |i_n Z_s(\omega)|^2 + 4k_B T \Re e[Z_s(\omega)]$$

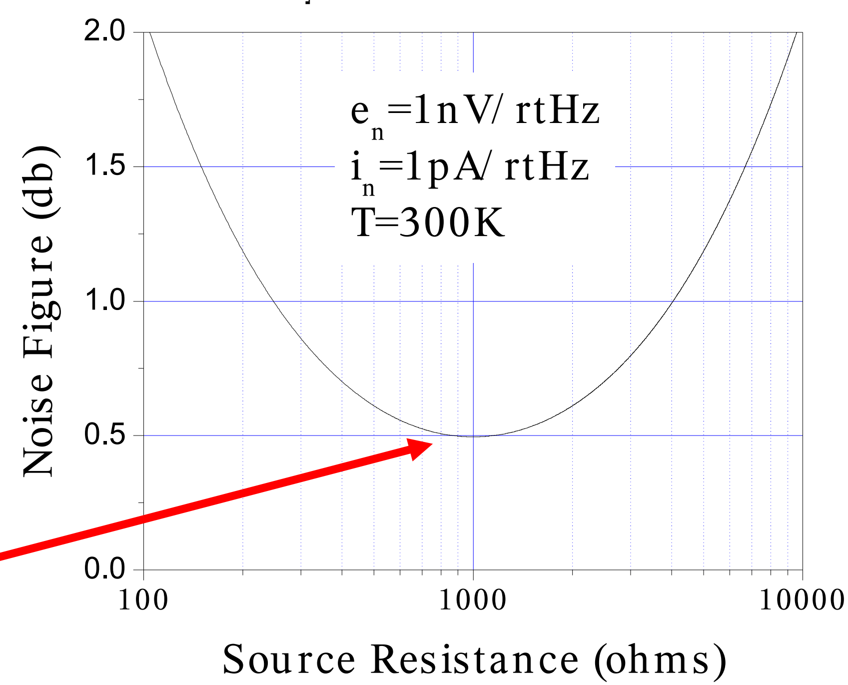
Define the noise figure as the ratio of the total noise to the thermal noise:

$$NF = 10\,Log\left[\frac{V_T^2}{4k_B T \Re e[Z_s(\omega)]}\right] = 10\,Log\left[\frac{e_n^2 + |i_n Z_s(\omega)|^2}{4k_B T \Re e[Z_s(\omega)]} + 1\right]$$

Assuming that $\boldsymbol{Z_s(\omega)=R}$ (pure resistive source, the noise figure minimizes when:

$$R = R_{opt} = \frac{e_n}{i_n}$$

$\boldsymbol{R_{opt}}$ is the amplifier "sweet spot"



Amplifier Noise Temperature

If we assume the source resistance to be at T=0, there still is noise at the output of the amplifier, this defines the noise temperature:

$$V^2 = e_n^2 + (i_n R_s)^2 = 4 k_b T_N R_s$$

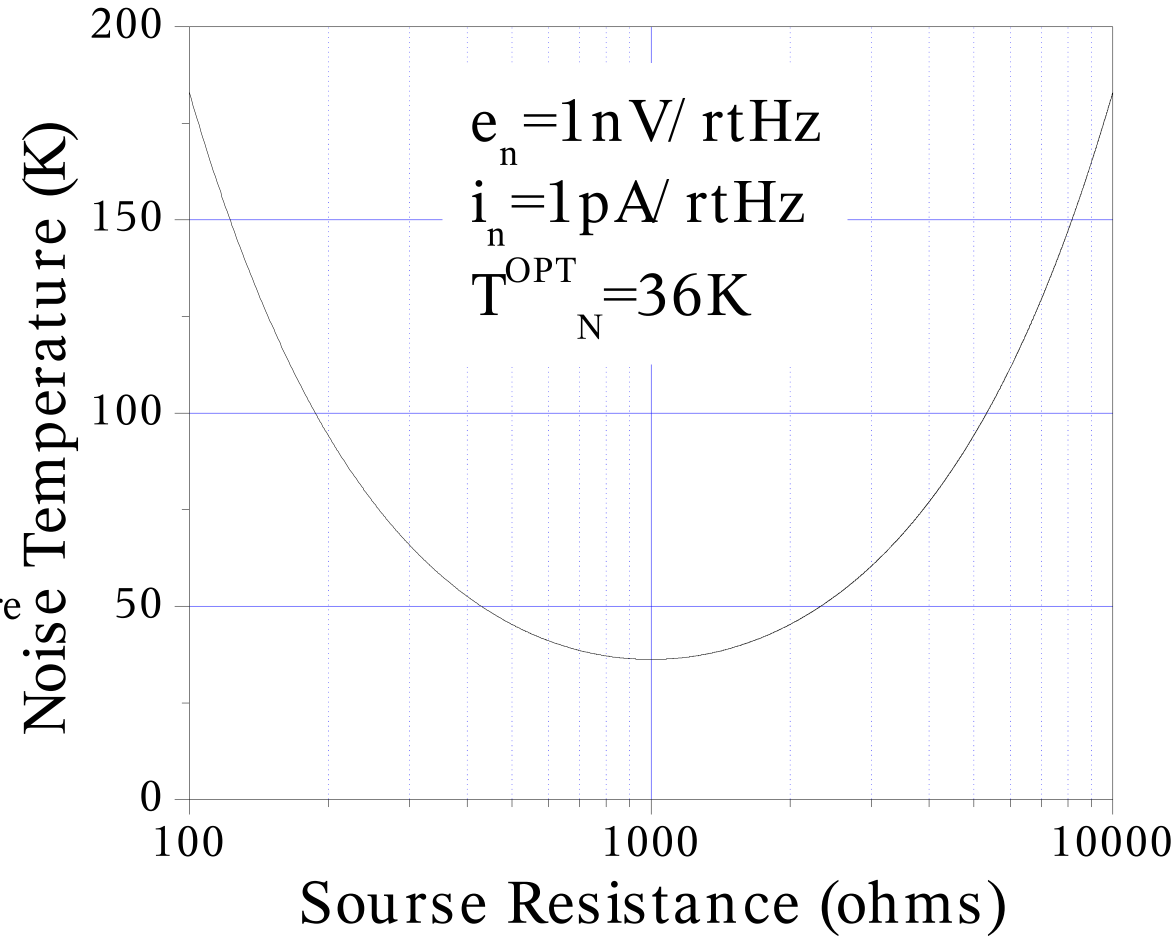
$$T_N = \frac{e_n^2 + (i_n R_s)^2}{4 k_b R_s}$$

Again, $\boldsymbol{T_N}$ can be minimized and the optimal source found:

$$R = R_{opt} = \frac{e_n}{i_n}$$

Which gives the lowest noise temperature

$$T_N^{OPT} = \frac{e_n i_n}{2 k_b}$$



Motion caused by Backaction

i_N integrated over the bandwidth of the oscillator will give a total current noise of:

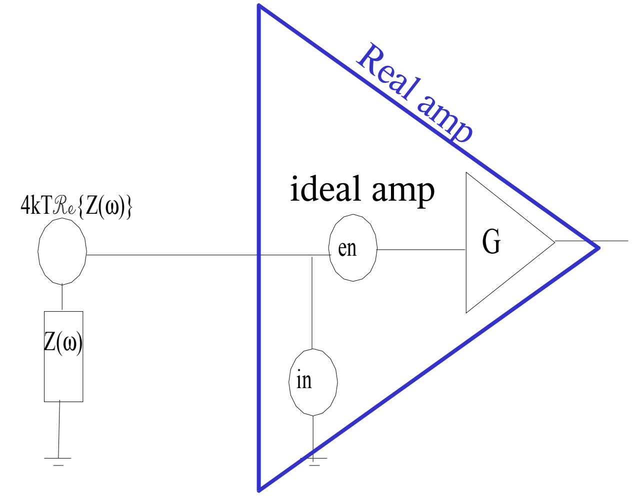
$$I_N = i_N \sqrt{\frac{\omega_0}{2\pi Q}}$$

I_N will generate a force:

$$F = B I_N l = B l i_N \sqrt{\frac{\omega_0}{2\pi Q}}$$

Which will generate motion:

$$X_M = Q\frac{F}{k} = \frac{B l i_N}{k}\sqrt{\frac{Q\omega_0}{2\pi}}$$



Which is equivalent to the resonator at a temperature of:

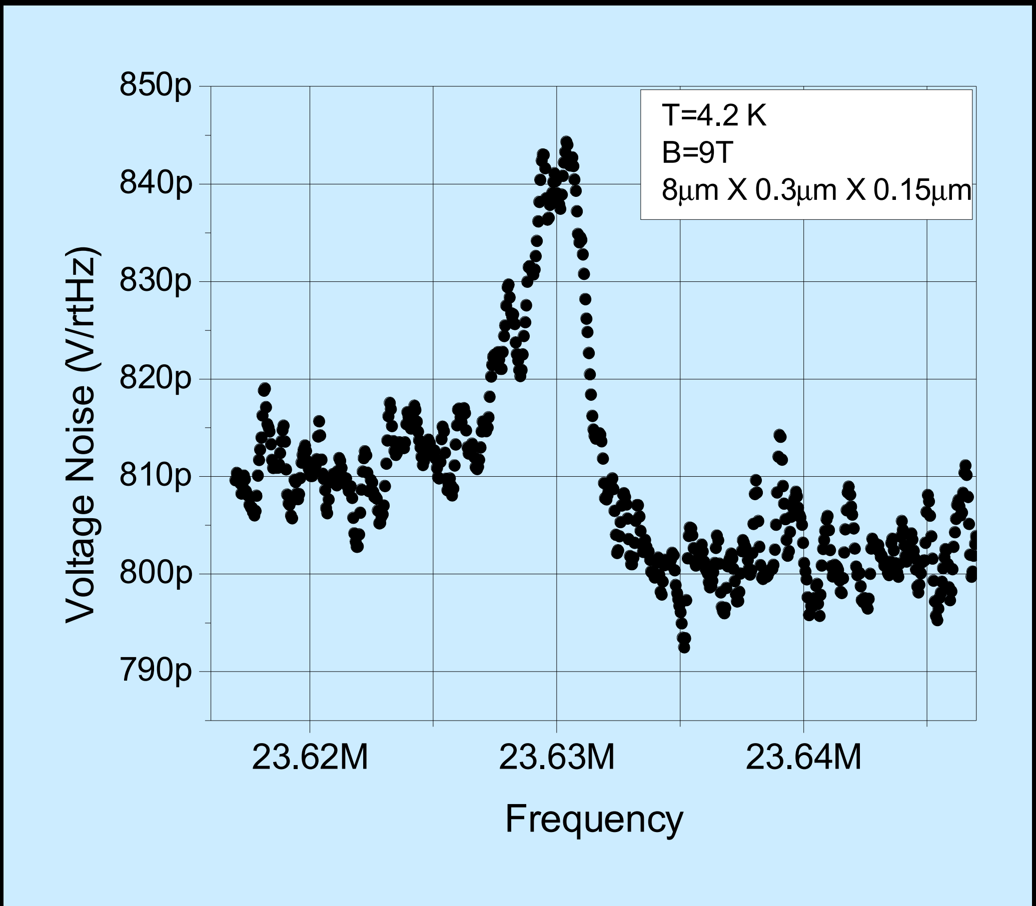
$$\frac{1}{2}k_B T_{BA} = \frac{1}{2}k X_M^2$$

$$T_{BA} = \frac{kX^2}{k_b} = \frac{Q(Bl)^2\omega_0}{k}\frac{i_N^2}{2\pi k_b}$$

For optimal coupling, resonator is driven to the noise temperature of the amplifier.

Can we detect the Thermal Motion?

Amorphous Silicon Nitride resonator



To approach quantum limits motion we need a quantum limited linear amplifier:

SQUIDs and SETs

Conditions

$f_0 = 23MHz$

$Q = 10^4$

$T = 4.2K$

$B=9T$

$T_N=40K$

Thermomechanical Noise

$$\frac{1}{2}kx^2 = \frac{1}{2}k_B T$$

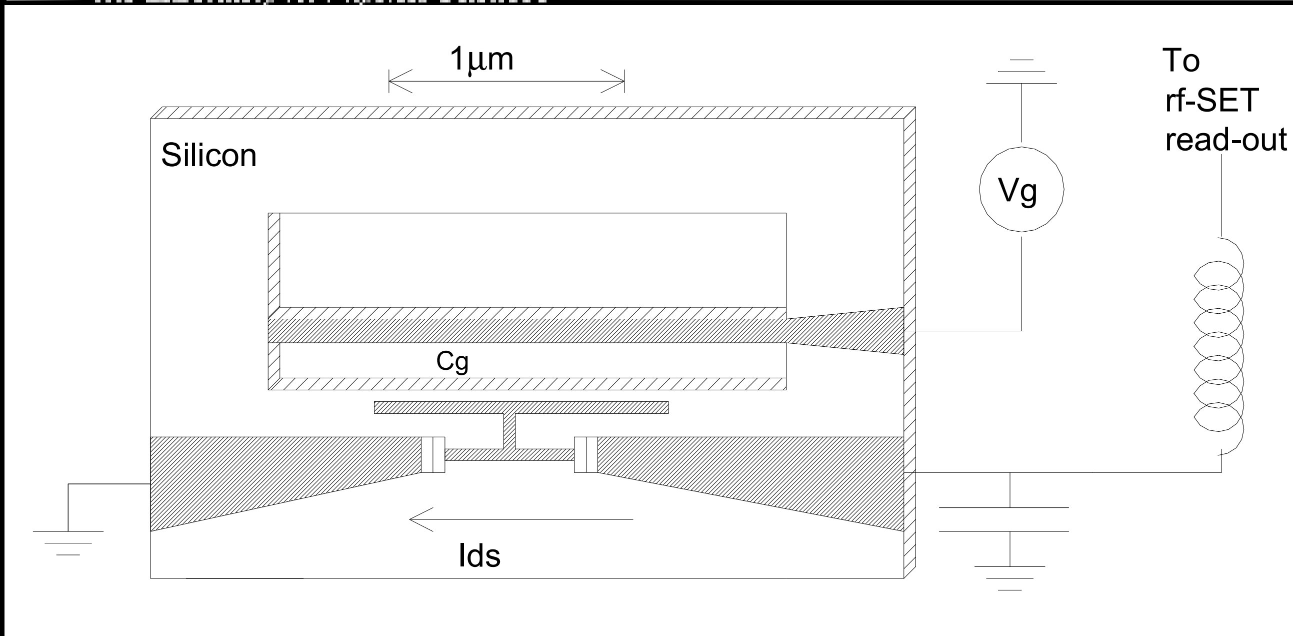
$x_{RMS} = 2\ pm_{RMS} \Rightarrow 8\ nV_{RMS}$

X_{RMS} (4.2K)= 1.4 pm= 60 X_{SQL}

Position detection with Single Electron Transistors…

quantum mechanically perfect amplifiers for NEMS

SET Position Measurement



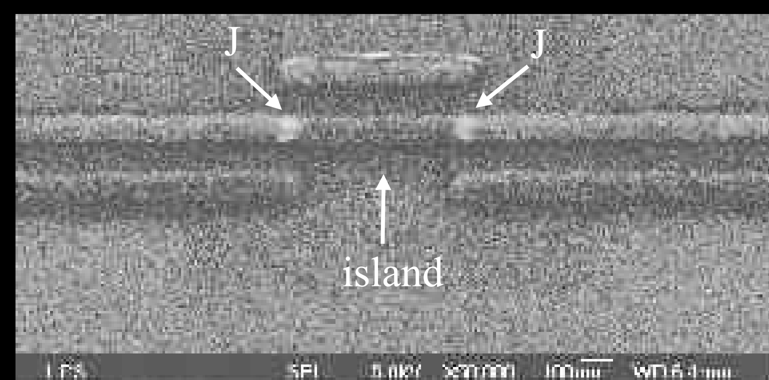
Energy sensitivity of rf SET has been demonstrated to be ~4.8 $\hbar$

(not including back action)

Assime APL 2001.

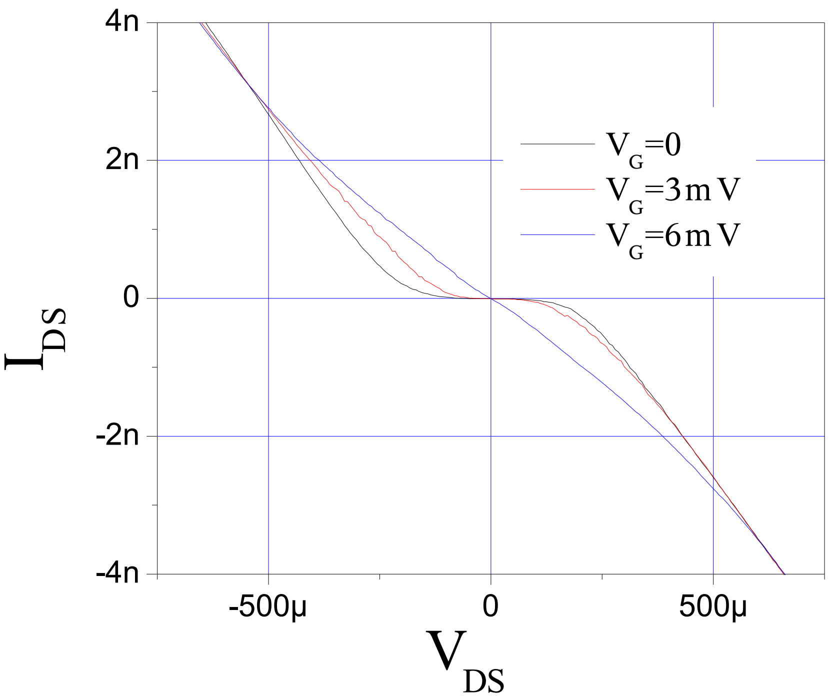
Coupling: $I_{DS}\left(V_{DS}, C_g(x) V_g\right)$

$$\delta x \Rightarrow \delta C_g \Rightarrow \delta I_{DS}$$



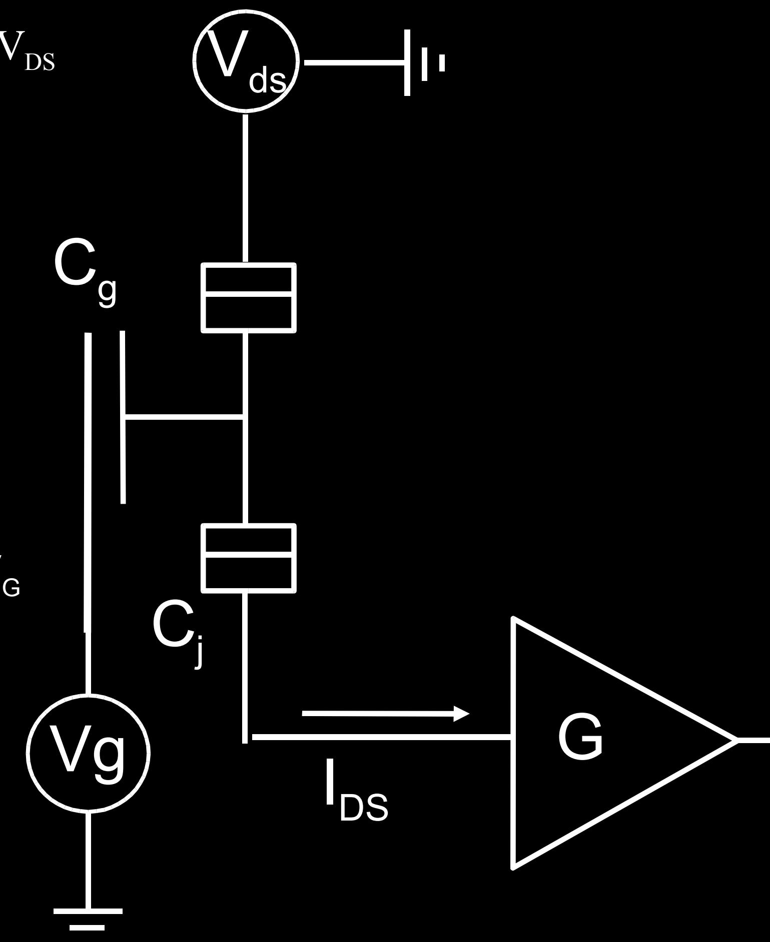
Blencowe and Wybourne, APL 77, 3845 (2000)

SET IV's and Modulation



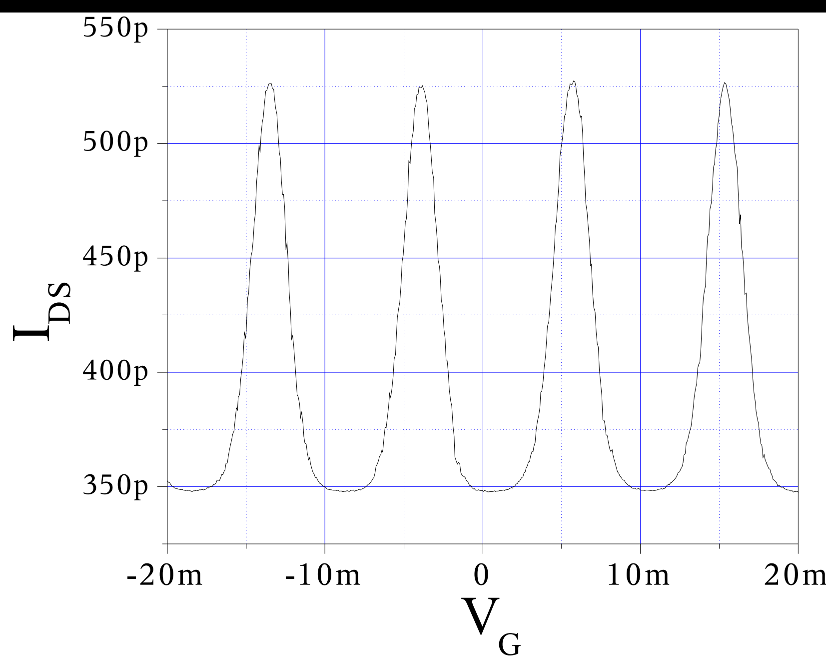
Fix V_G,

measure I_{DS} vs V_{DS}

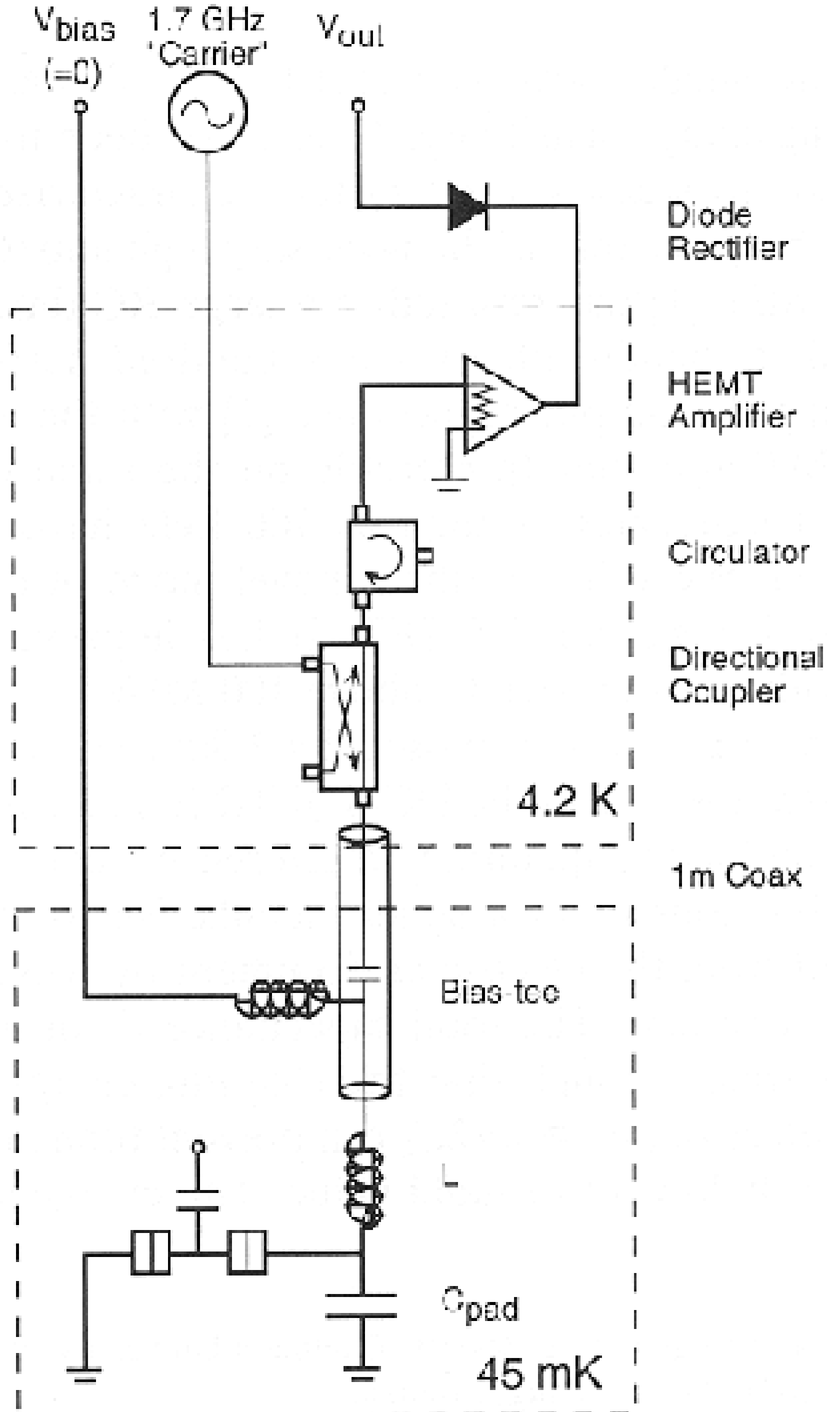


Fix V_{DS},

measure I_{DS} vs V_G



Conductance of SET is depends upon applied charge



rf SET-Ideal Amplifier for Nanomechanics

Impedance of SET is monitored by measuring microwave reflections

Microwave tank transforms ~**50kΩ** impedance to ~**50Ω**

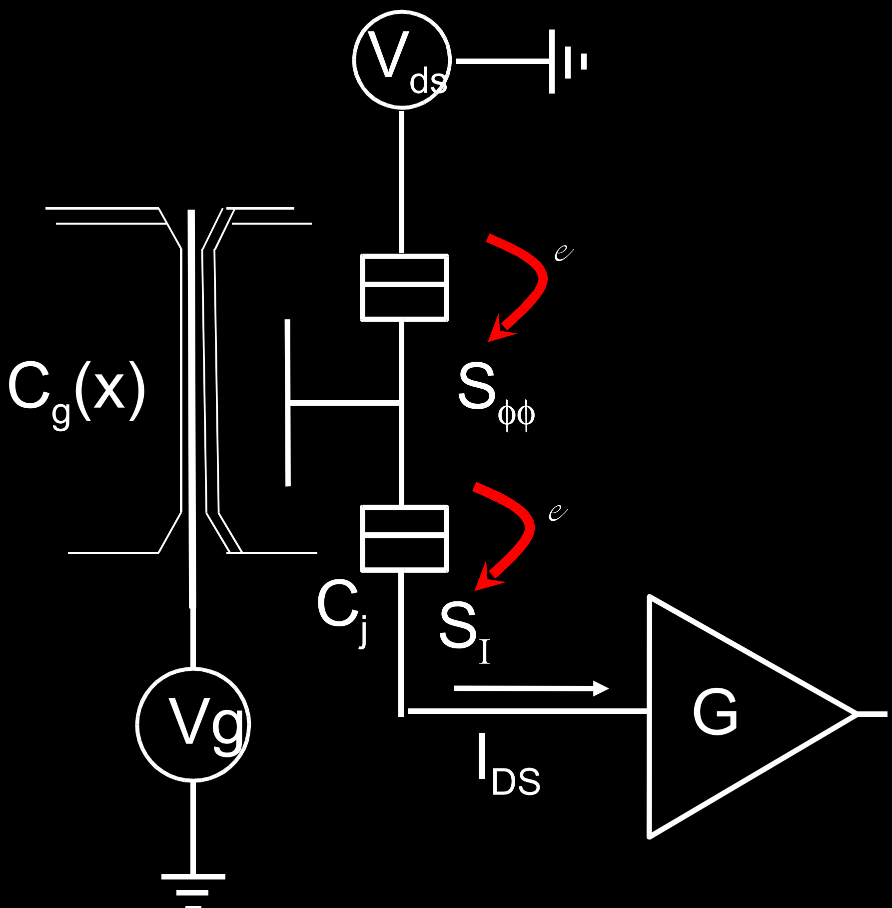
Charge sensitivity:	~5 μ e/ rtHz
Band width:	75 MHz
1/f knee	10,000 Hz

Schoelkopf, et al, *Science* **280**, 1238 (1998).

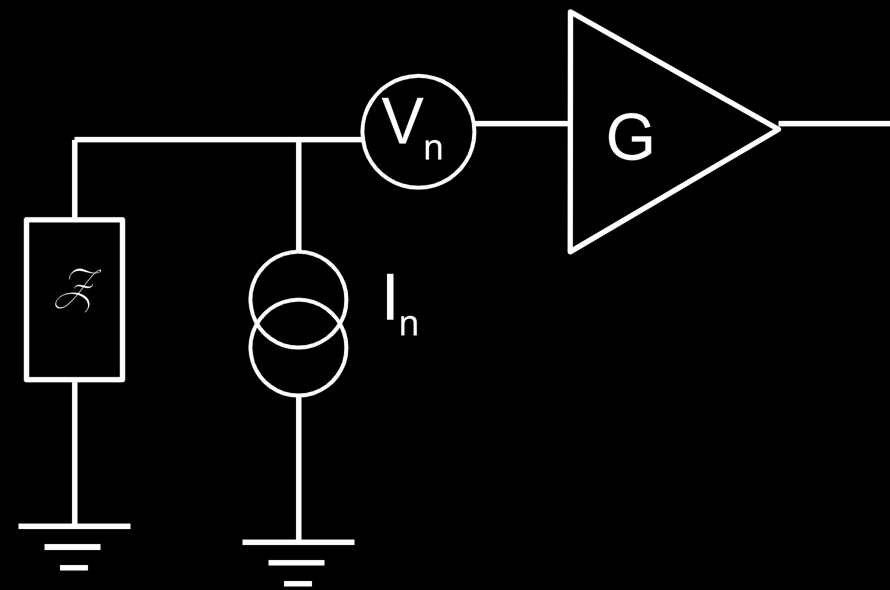
Wahlgreen, el al, *J. Supercond.* **12**, 741 (1999).

Back-action from SET

Electro-Mechanical Circuit

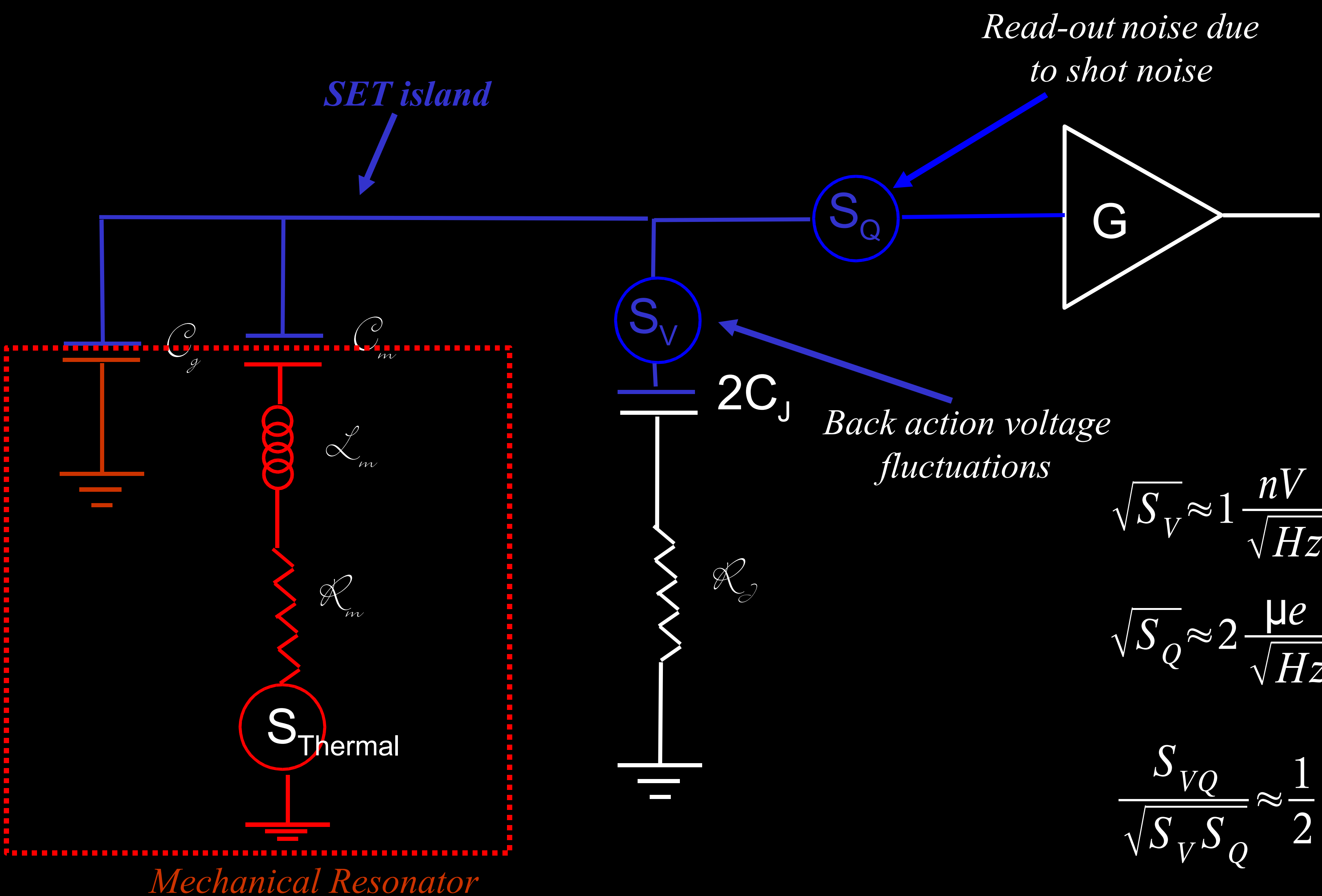


Noise in Electrical Circuits



Korotkov, PRB (1994).

Complete Noise Model

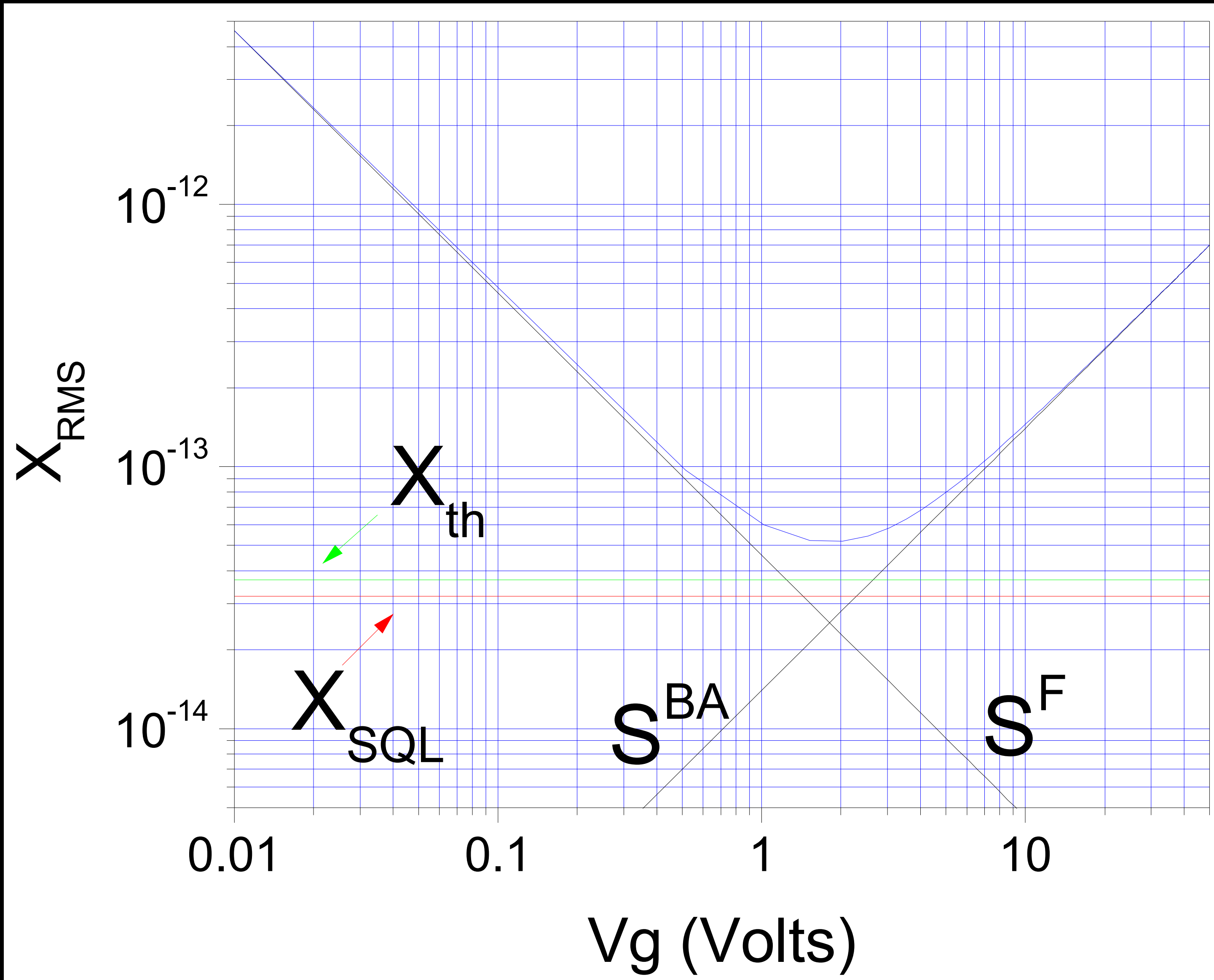


$$\sqrt{S_V} \approx 1 \frac{nV}{\sqrt{Hz}}$$

$$\sqrt{S_Q} \approx 2 \frac{\mu e}{\sqrt{Hz}}$$

$$\frac{S_{VQ}}{\sqrt{S_V S_Q}} \approx \frac{1}{2}$$

Displacement Resolution at the "Sweet Spot"



$¿ 10^{-15} \frac{m}{\sqrt{Hz}}$

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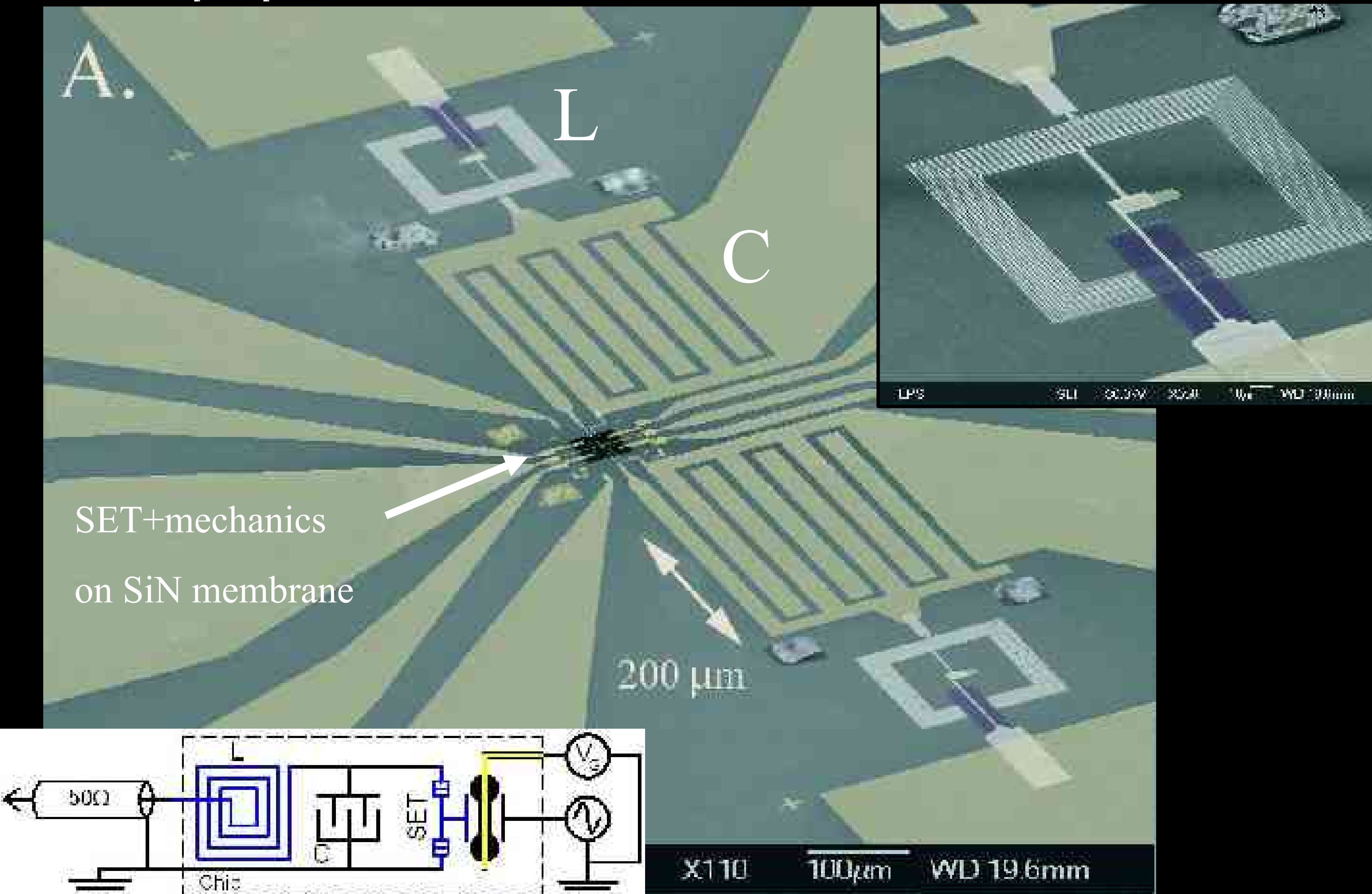
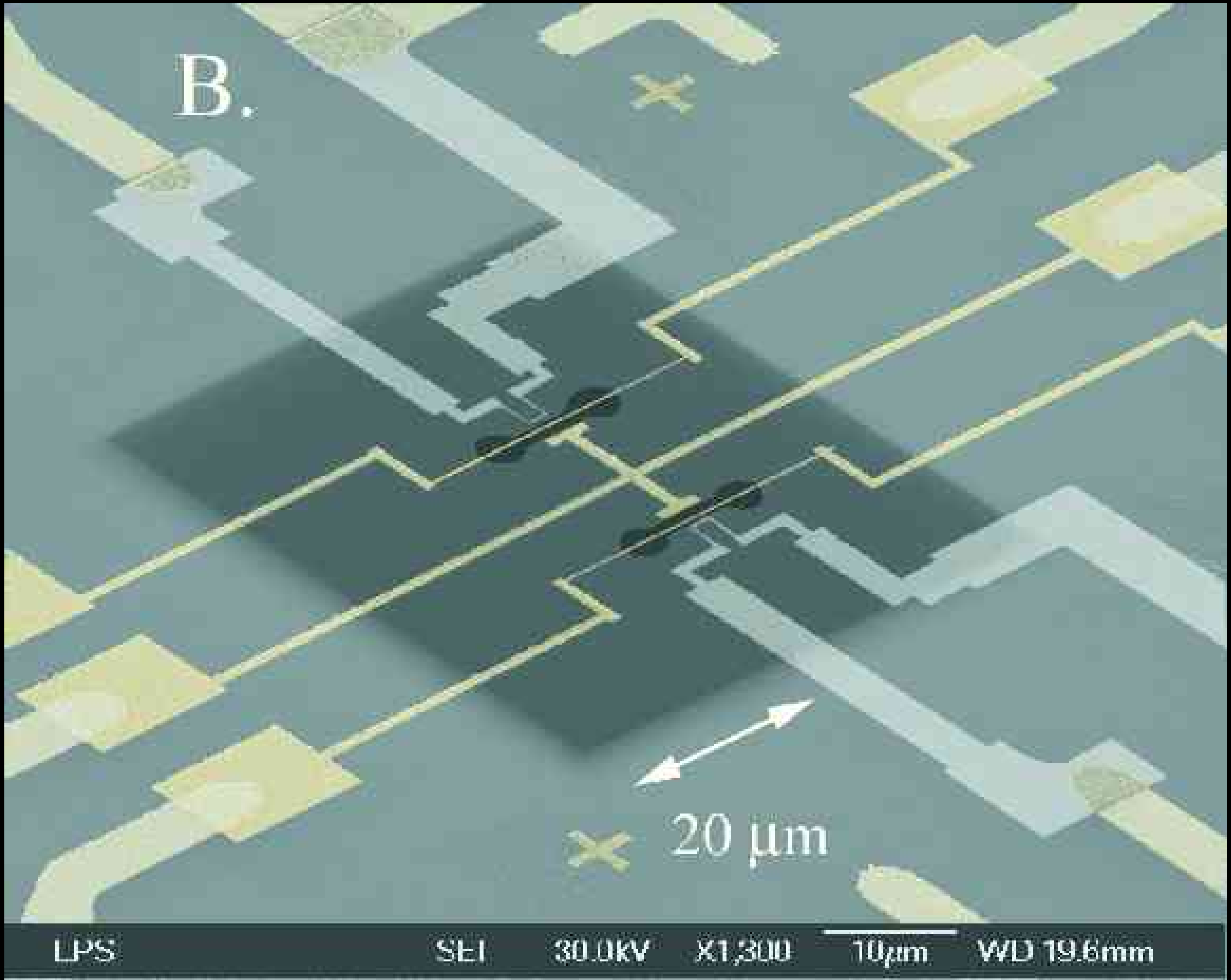
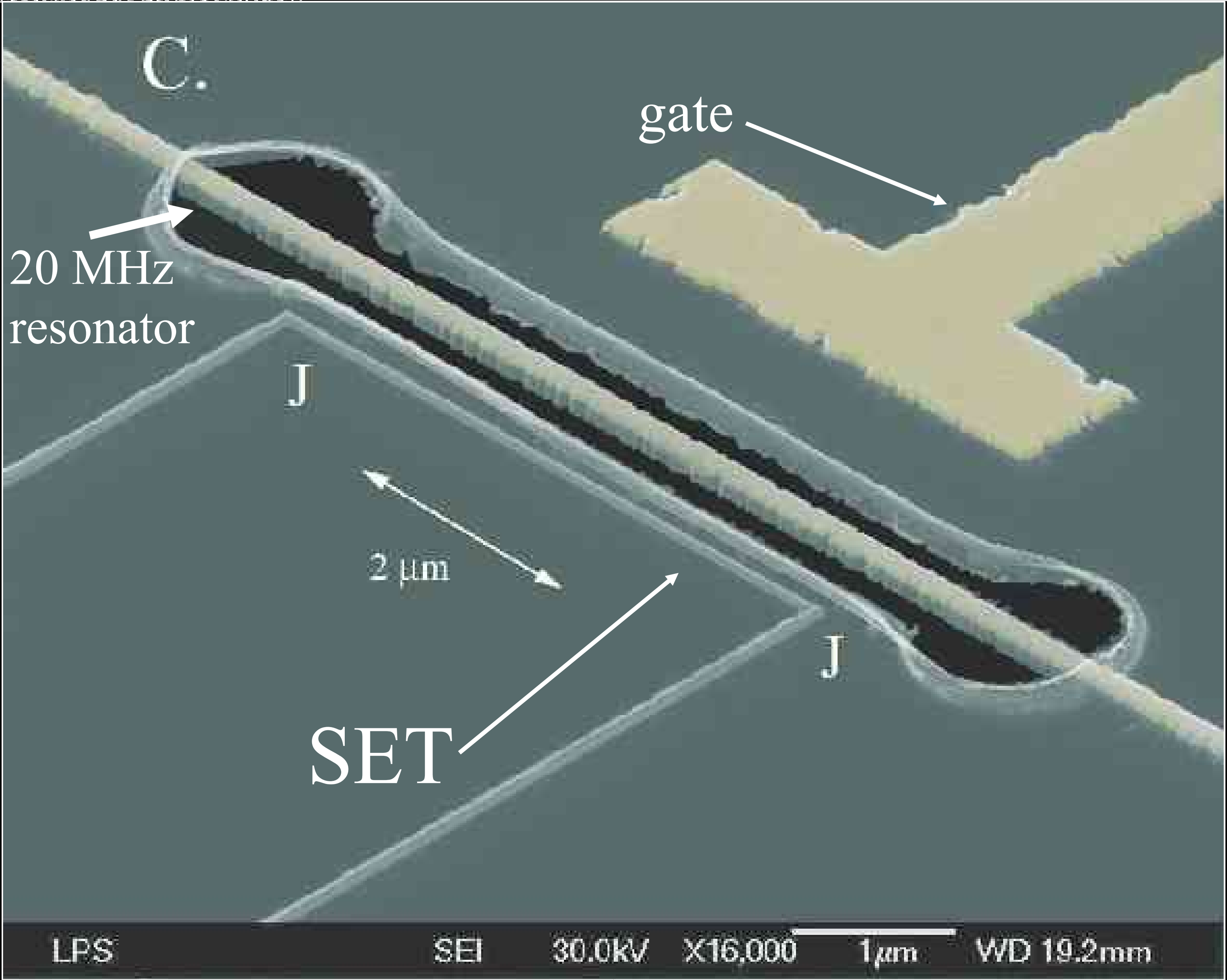
Recent Devices-RF SET coupled to Nanomechanics



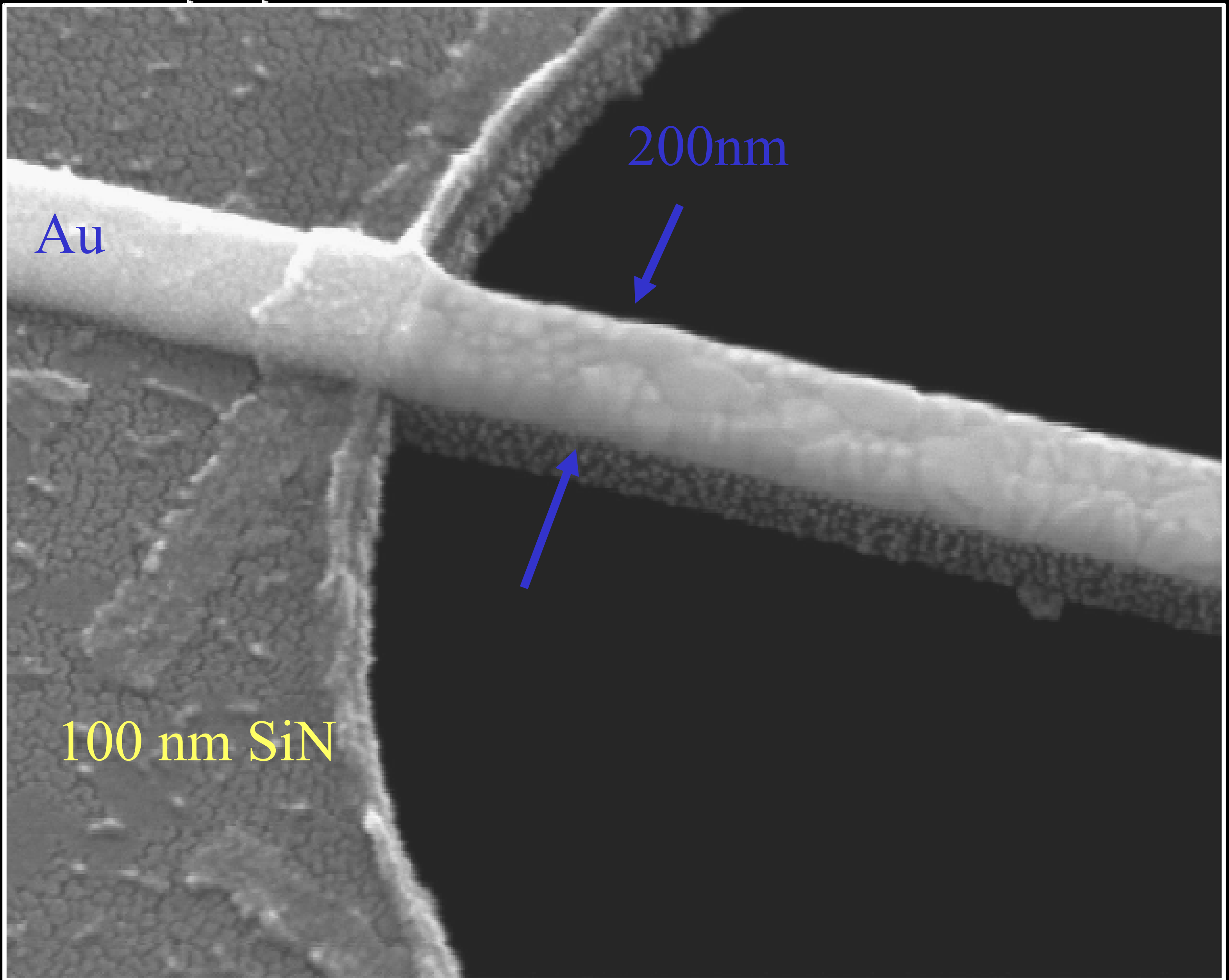
Recent Devices-RF SET coupled to Nanomechanics



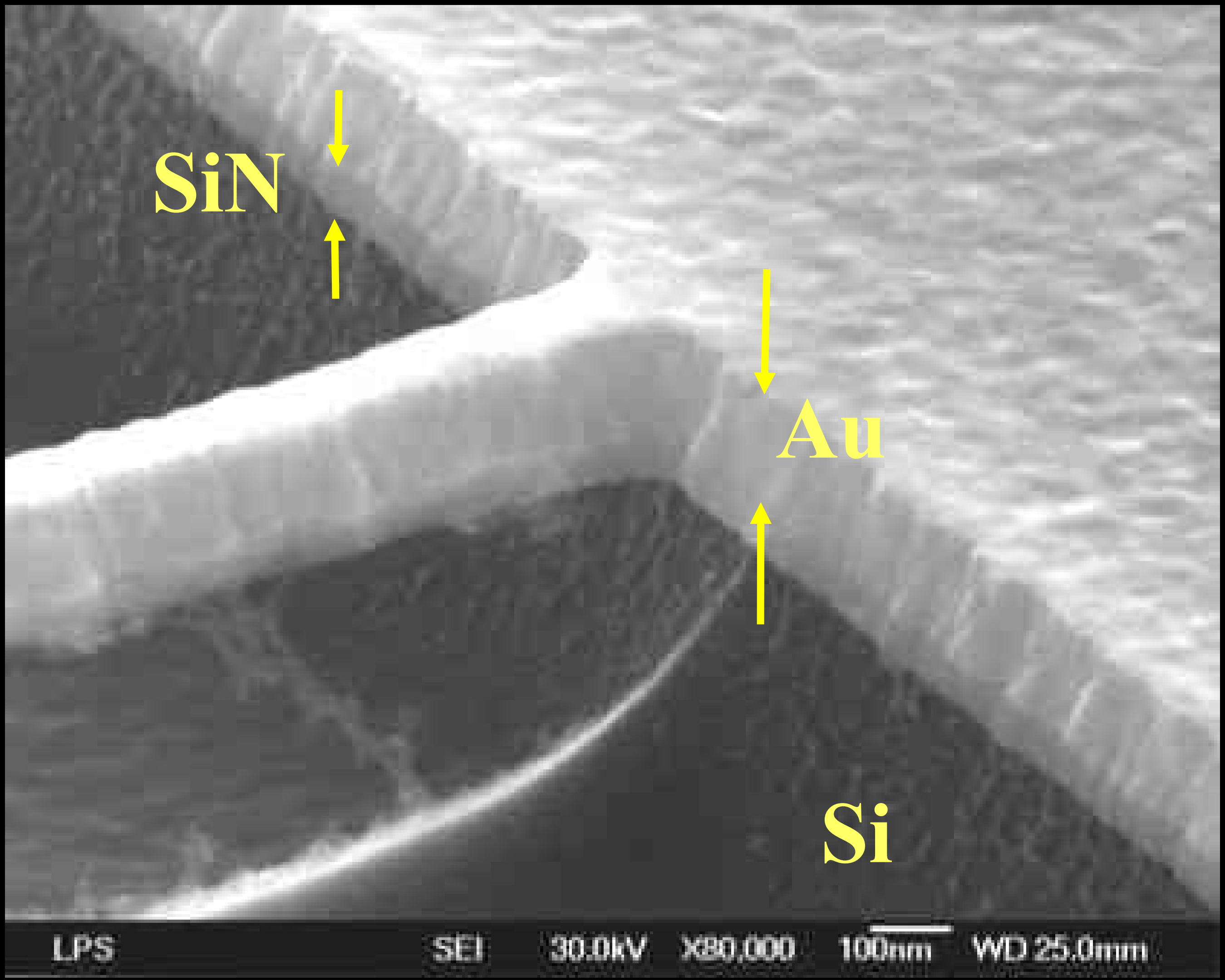
Recent Devices-RF SET coupled to Nanomechanics



Recent Devices-RF SET coupled to Nanomechanics



Improvements to fabrication



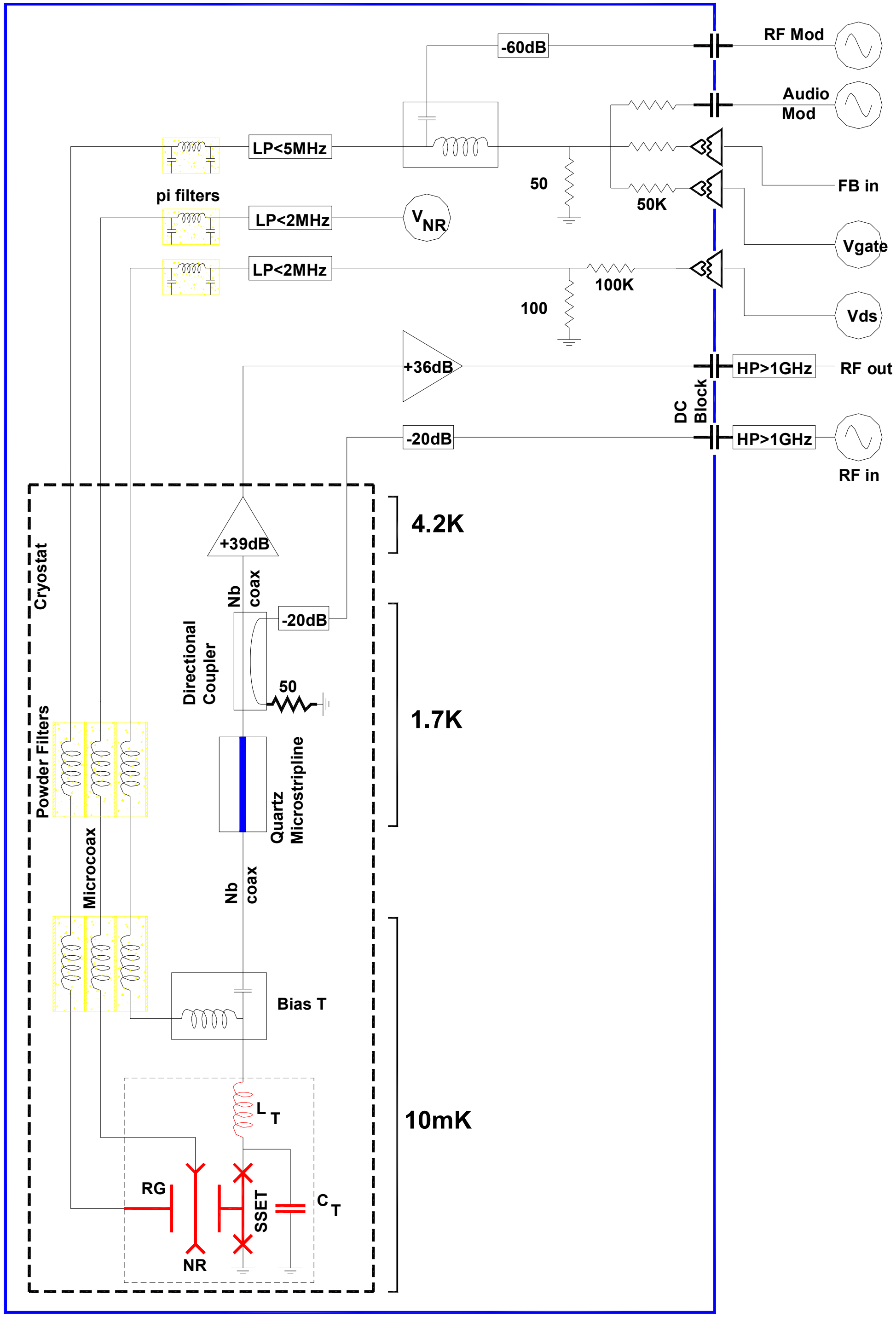
Use dry process to undercut

*no membrane

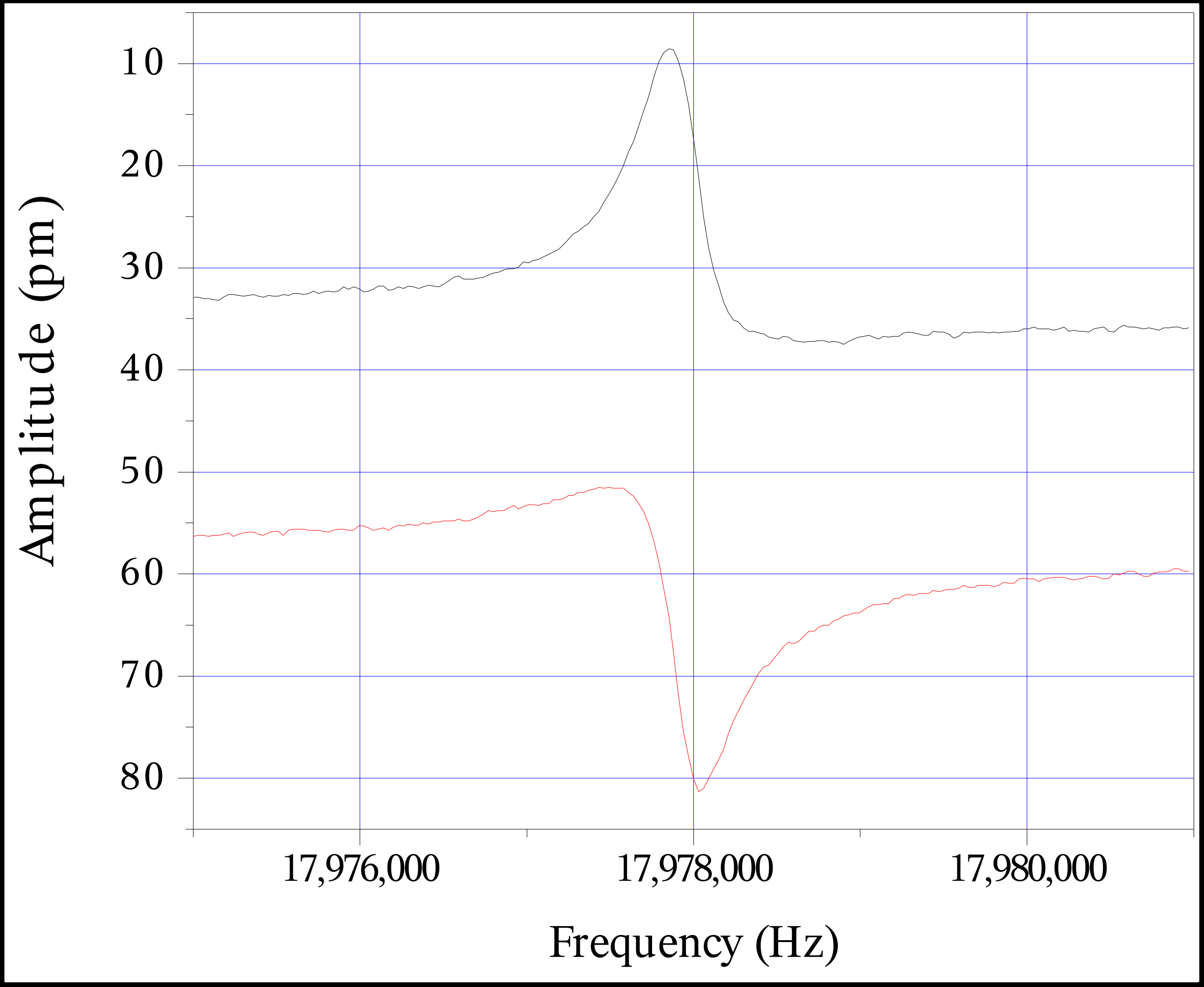
Move all photo lithography to foundry

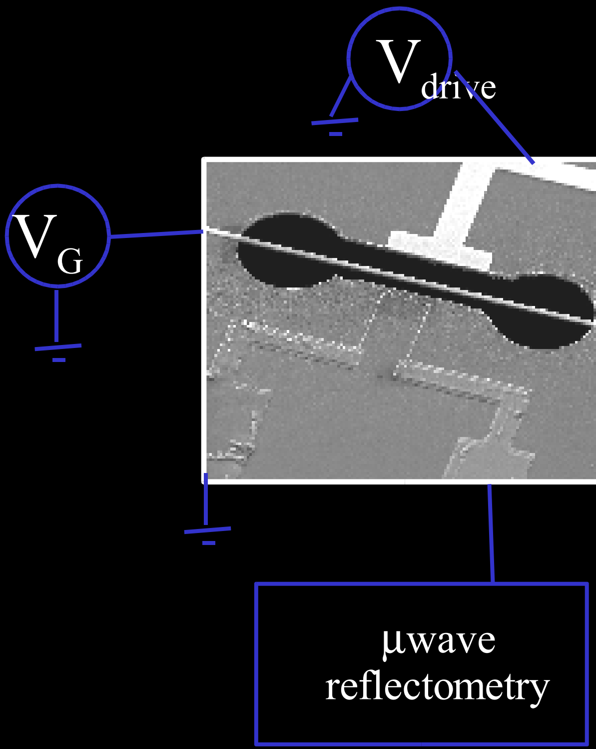
*saves 4 layers of litho

Cryogenic and electronic setup



Resonance Detection with rf SET



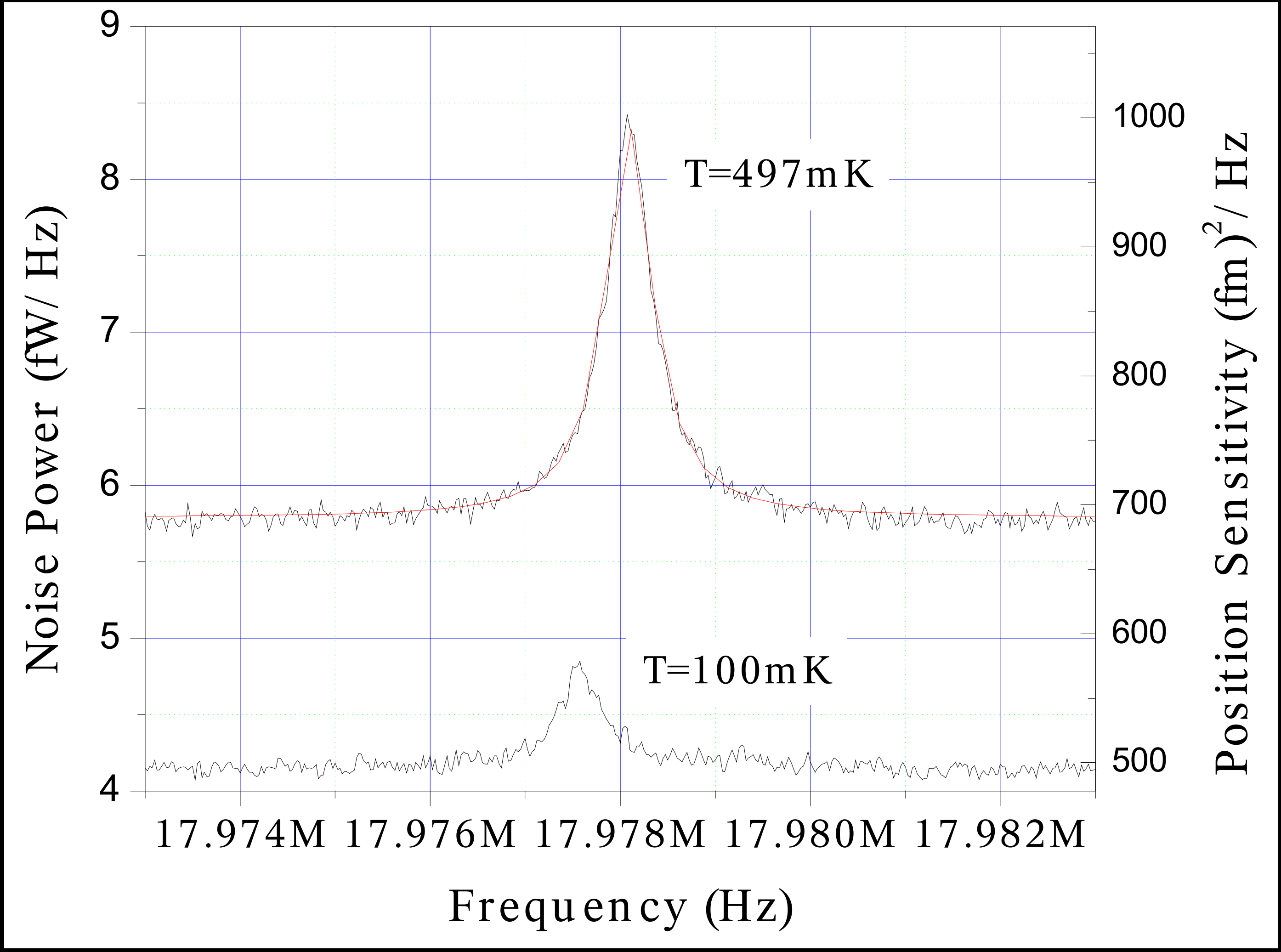


Mechanical resonance detected by electrostatically driving beam.

Thermal Motion of Nanomechanics

Equipartion Theorem: $\frac{1}{2} k_B T = \frac{1}{2} m\omega^2 x_{RMS}^2$

$V_G = 2V$



Noise Temperature of our system with V_G=2V:

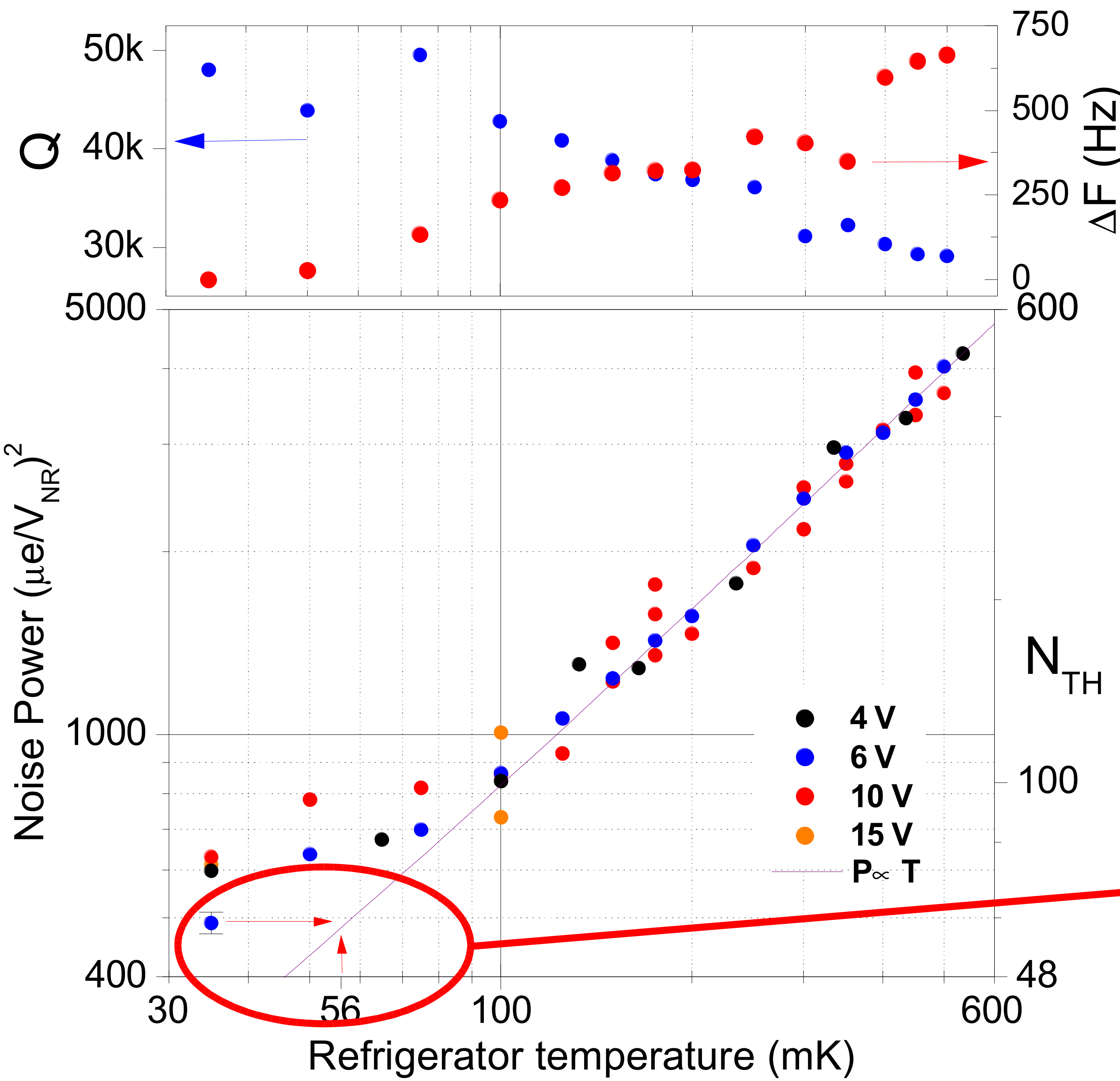
$T_N \sim 1K$

Thermal Motion:

5.4 $10^{-13} m_{RMS}$ @ 500mK

2.4 $10^{-13} m_{RMS}$ @ 100mK

Nanomechanical Noise Thermometry



$T \approx 56\,mK$

$N_{TH} \approx 58$

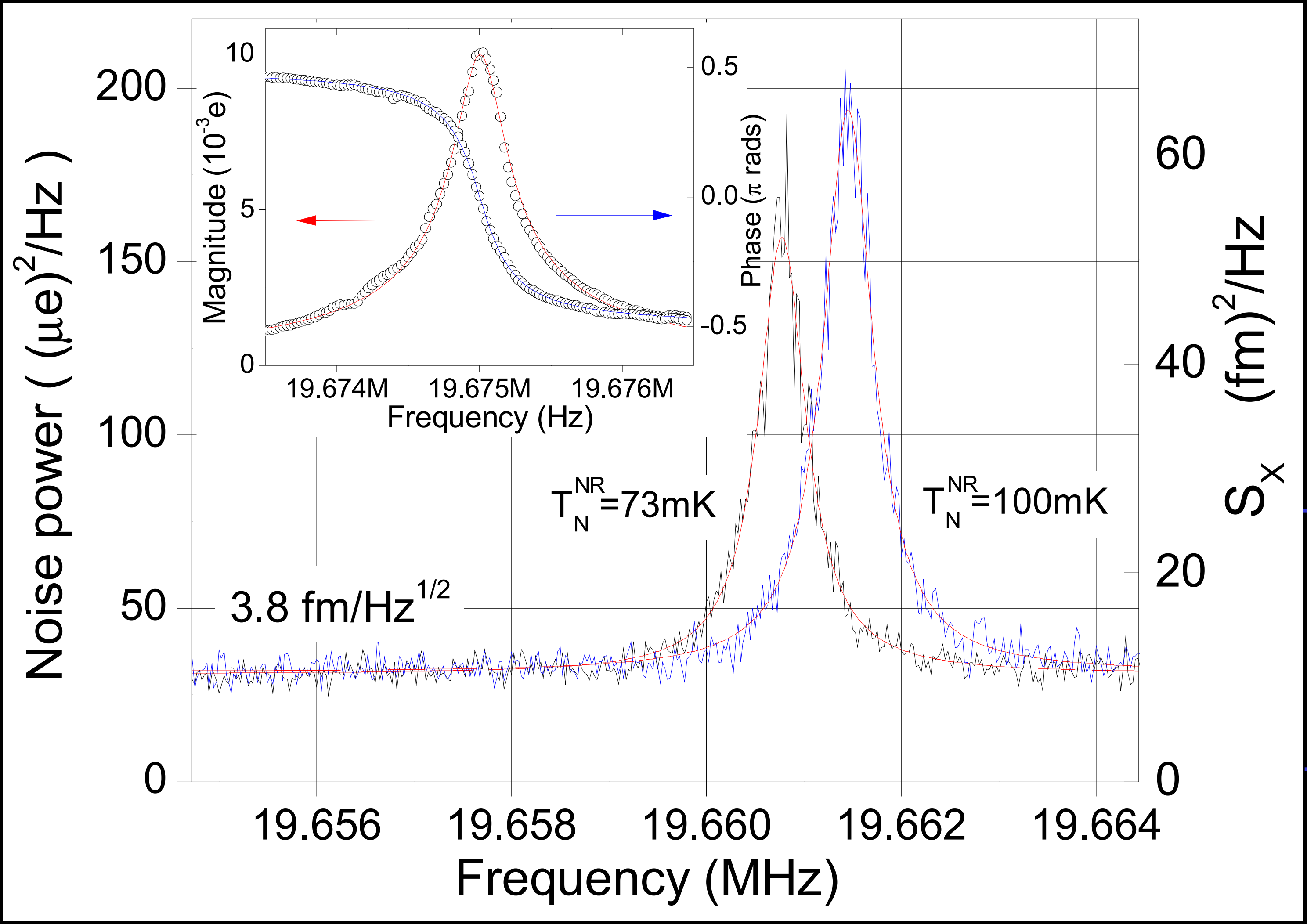
This is the closest approach to freeze-out to date of any mechanical resonator.

Detection Noise Temperature

$$\Delta x_{SQL} = \sqrt{\frac{\hbar}{2 m_{eff}\,\omega}} = 19\ fm$$

$$\Delta x_{QL} = \sqrt{\frac{\hbar}{\ln 3\, m_{eff}\,\omega}} = 26\ fm$$

$$\Delta x_{Noise} = \sqrt{S_x \Delta f_N} = 123\ fm$$



$V_G = 15\,V$

$T_N = 16\,mK$

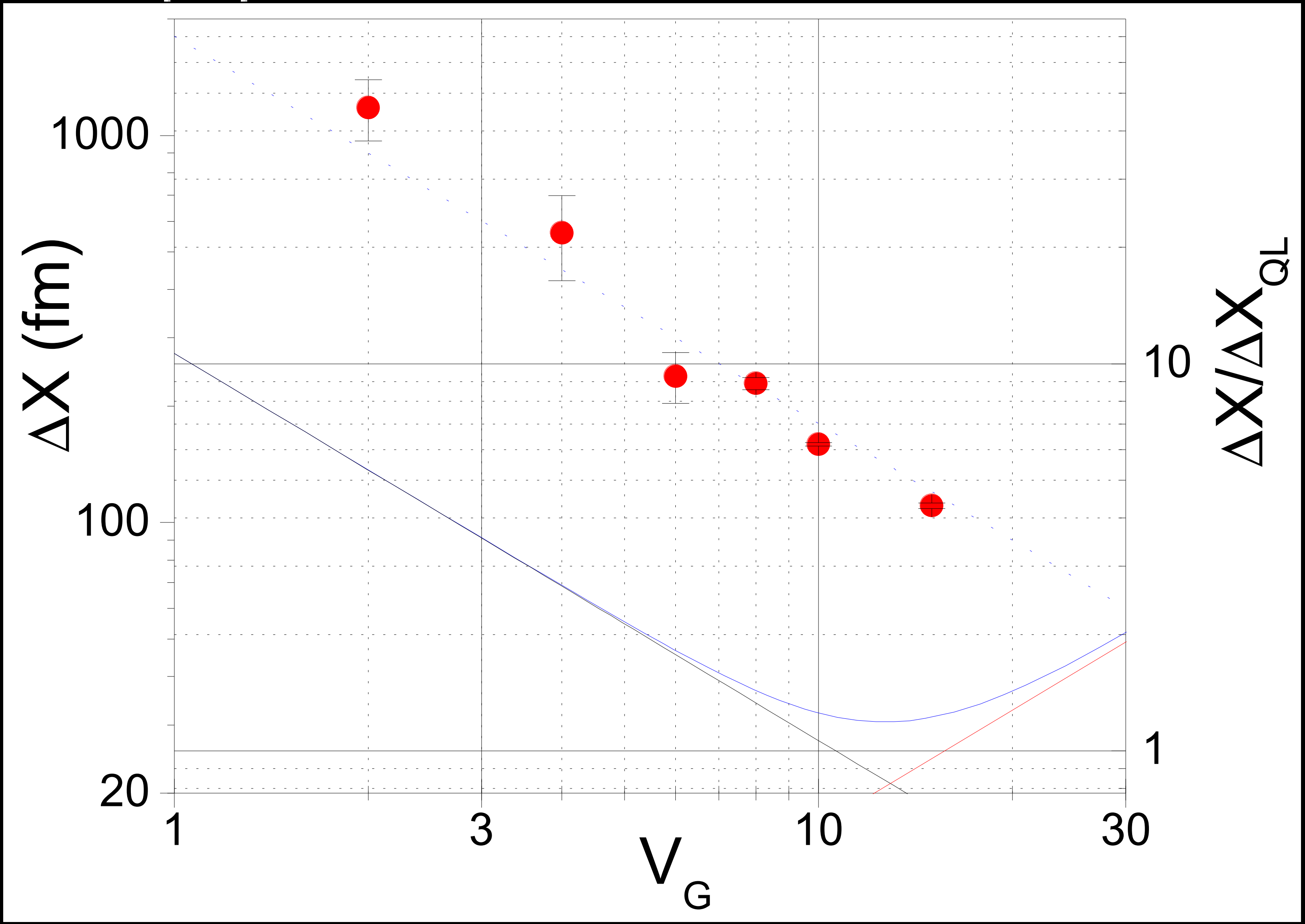
$T_N/T_Q = 18$

Knobel and Cleland (Nature, July 2003)

$T_N = 30K$

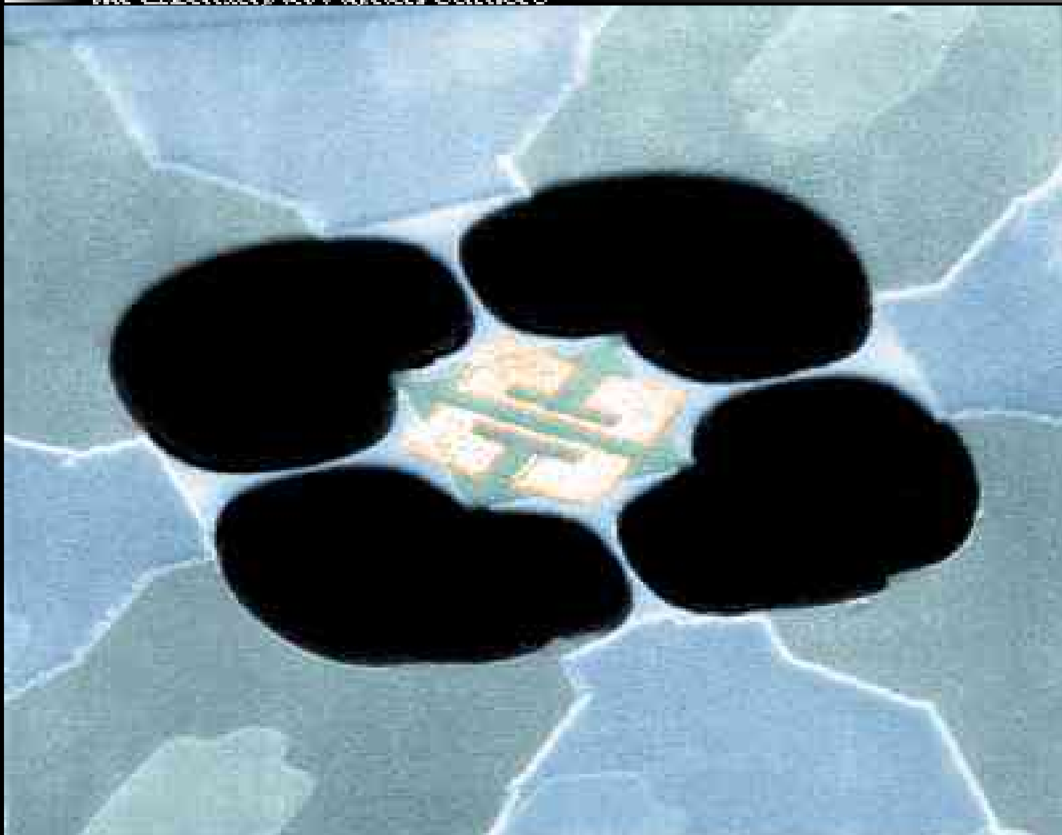
$T_N/T_Q = 6000$

Measured Displacement Resolution

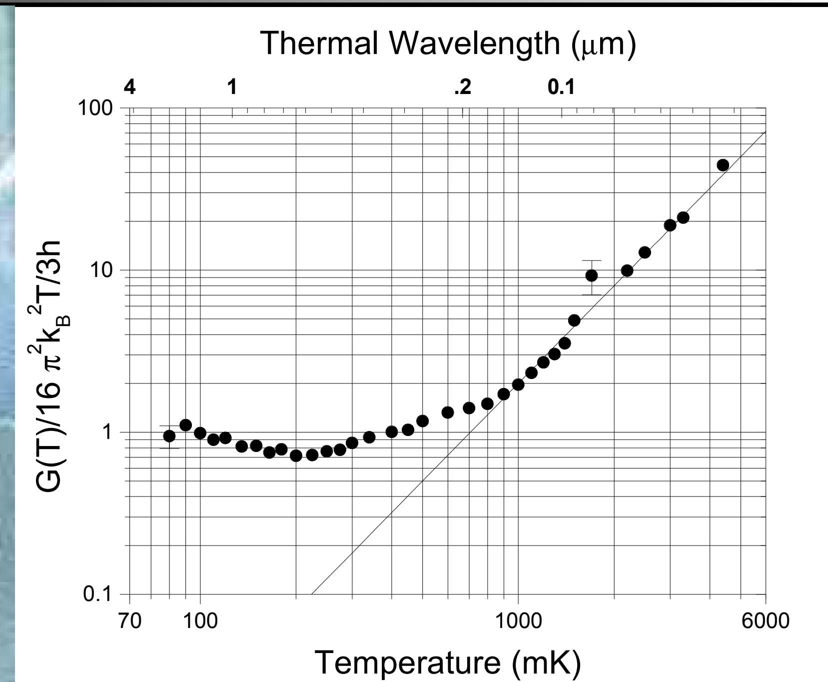


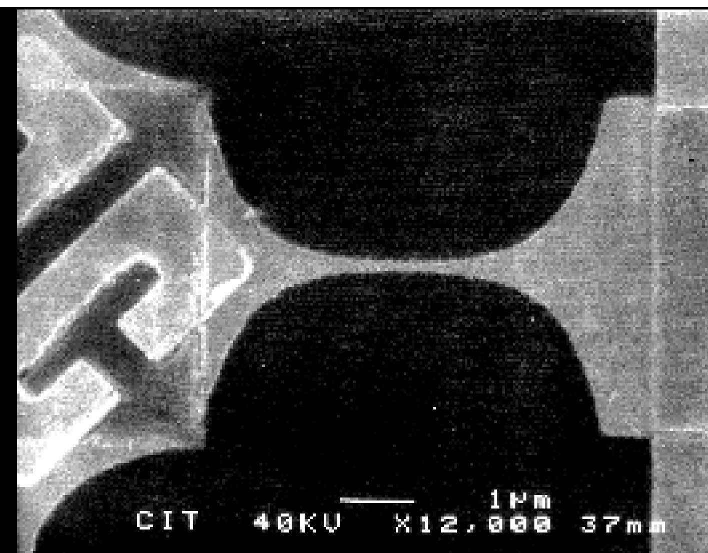
Science, 2 April 2004

Nanomechancis at Freeze-Out



freeze-out to 1D phonon channels



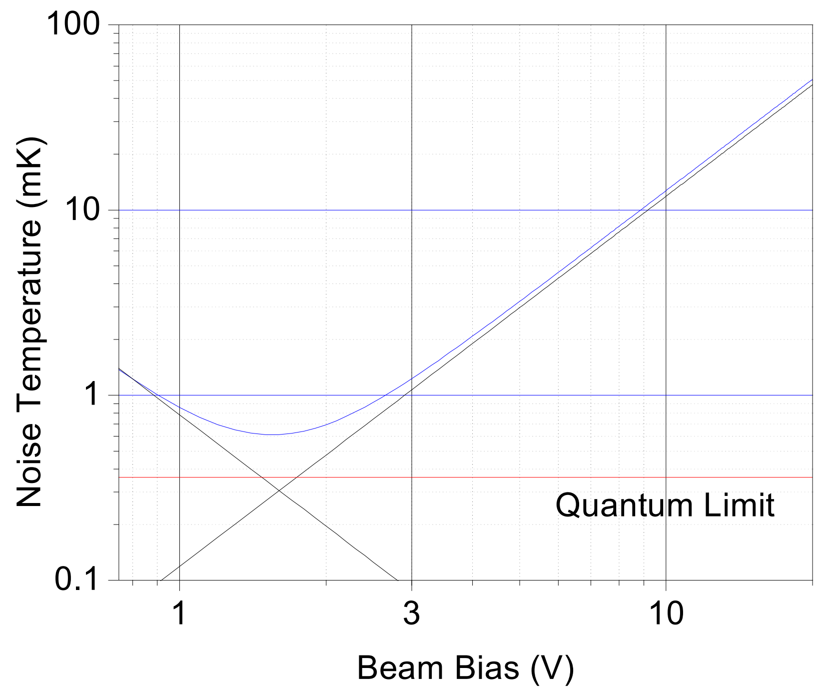


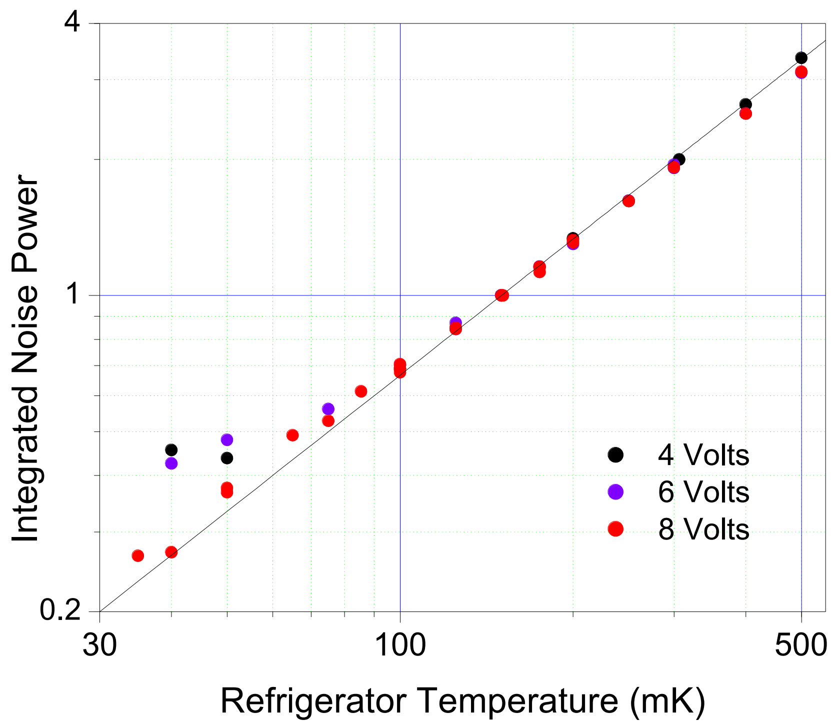
Measurement of the quantum of thermal conductance

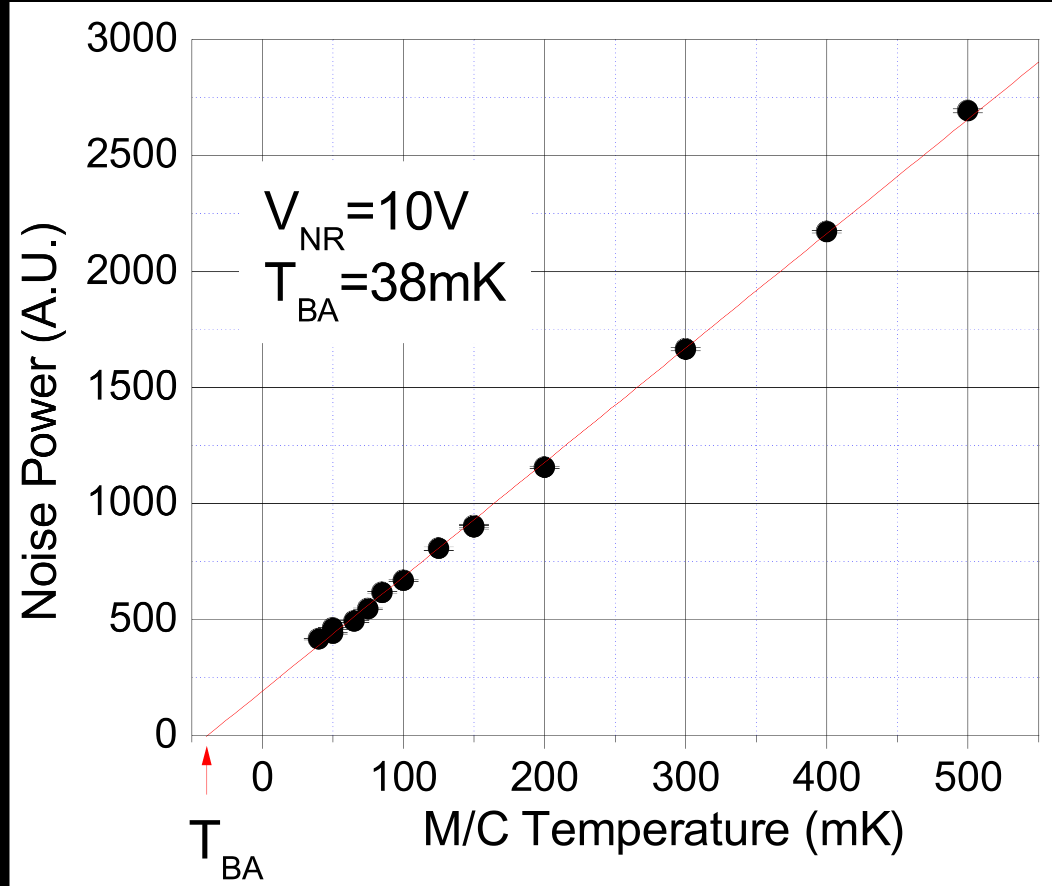
K. Schwab, E.A. Henriksen, J.M. Worlock & M.L. Roukes

NATURE|VOL 404|27 APRIL 2000

Strong Coupling Experiments

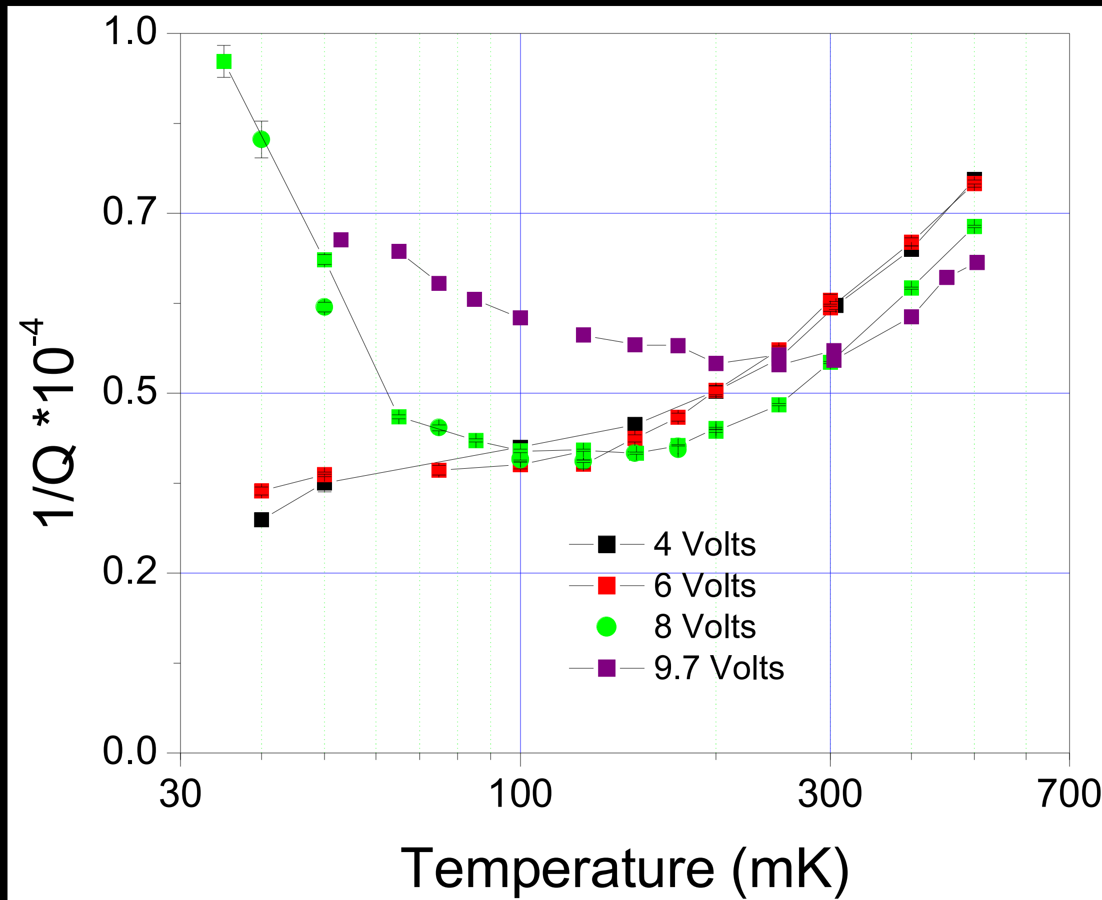






Classical dynamics of a nanomechanical resonator coupled to a single-electron transistor, A. D. Armour, M. P. Blencowe, and Y. Zhang, Phys. Rev. B **69**, 125313 (2004)

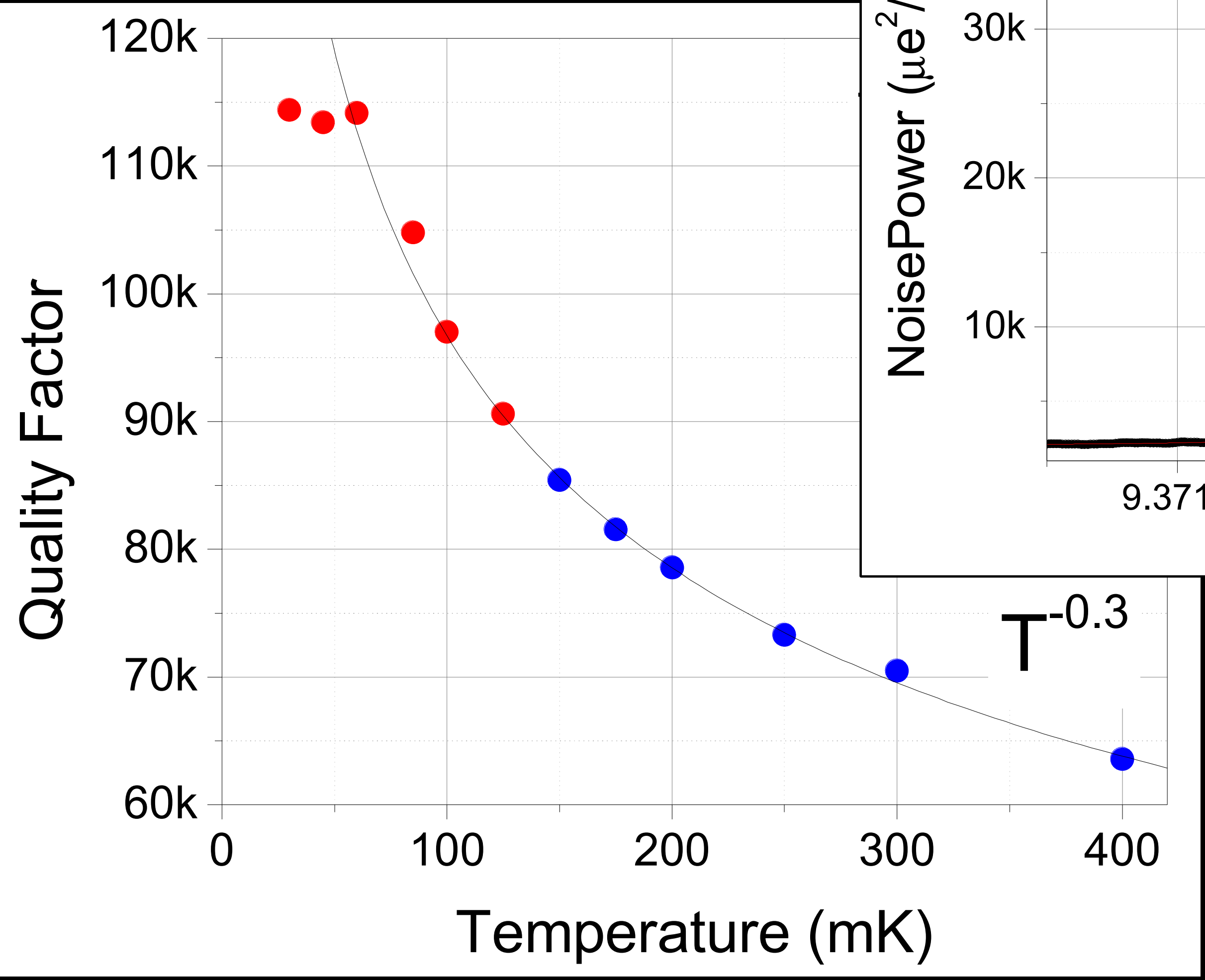
Strong Coupling Experiments

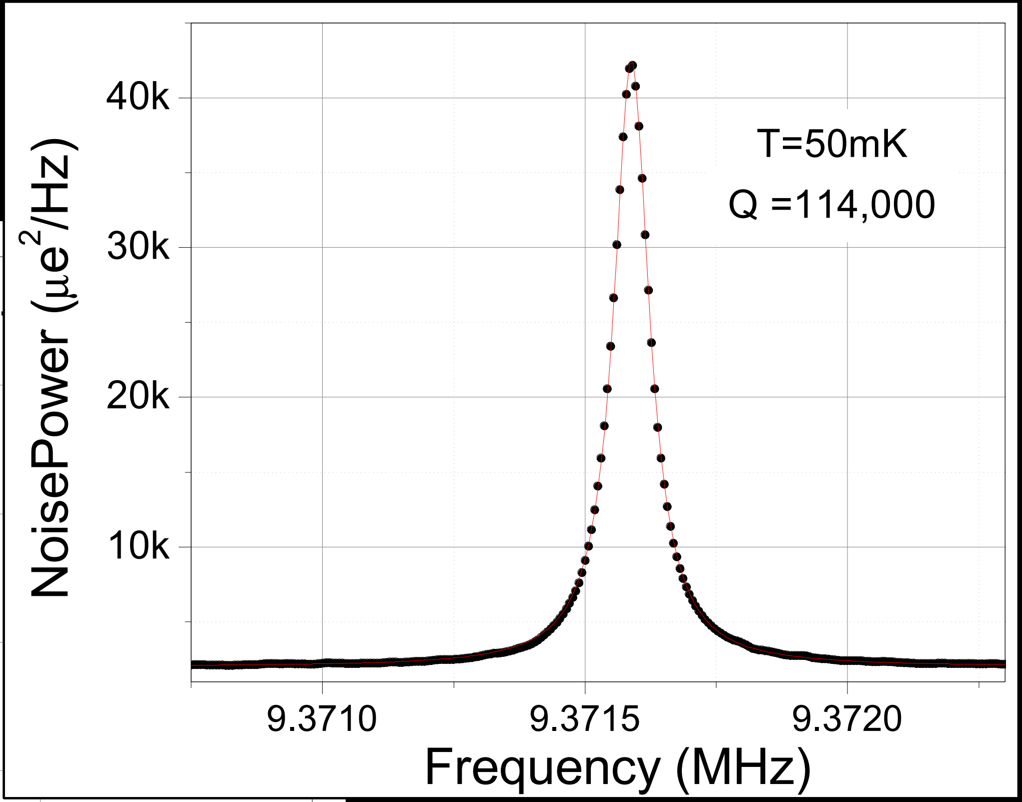


Phonon Bath $\frac{1}{Q_{Bath}}$ ⟷ Mechanical Mode $\frac{1}{Q_{SSET}}$ ⟷ SSET

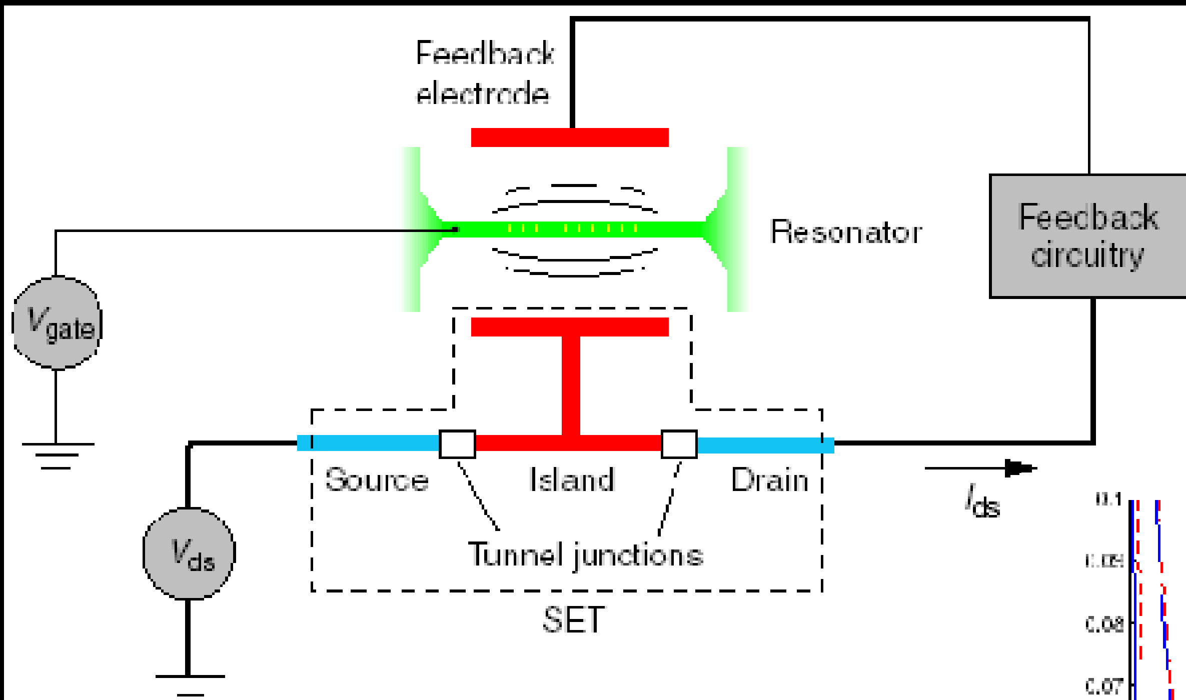
Classical dynamics of a nanomechanical resonator coupled to a single-electron transistor, A. D. Armour, M. P. Blencowe, and Y. Zhang, Phys. Rev. B **69**, 125313 (2004)

Q vs Temperature



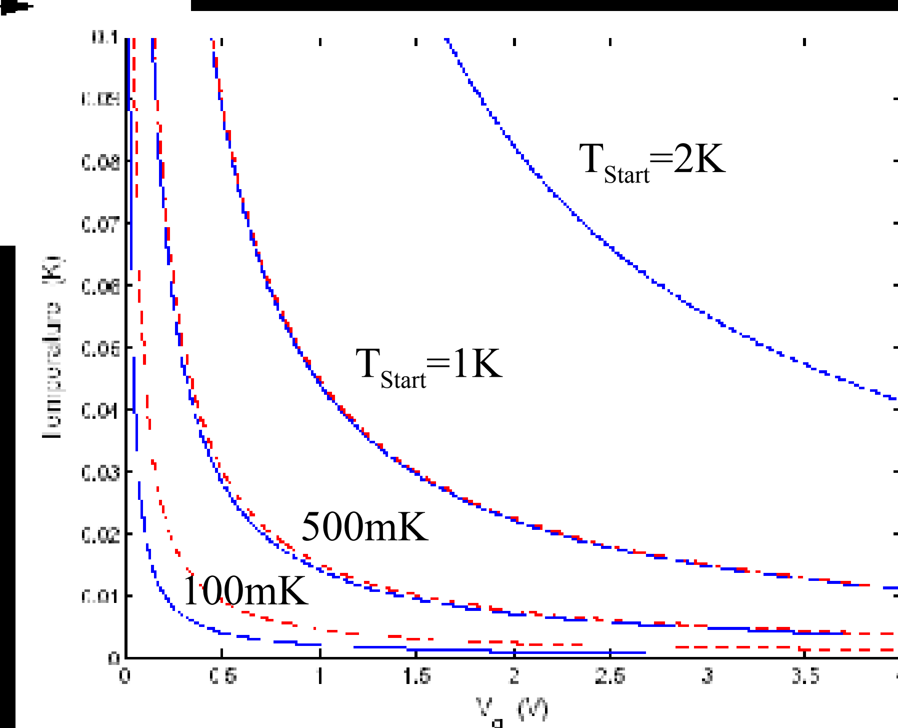


Active Cooling to the Ground State



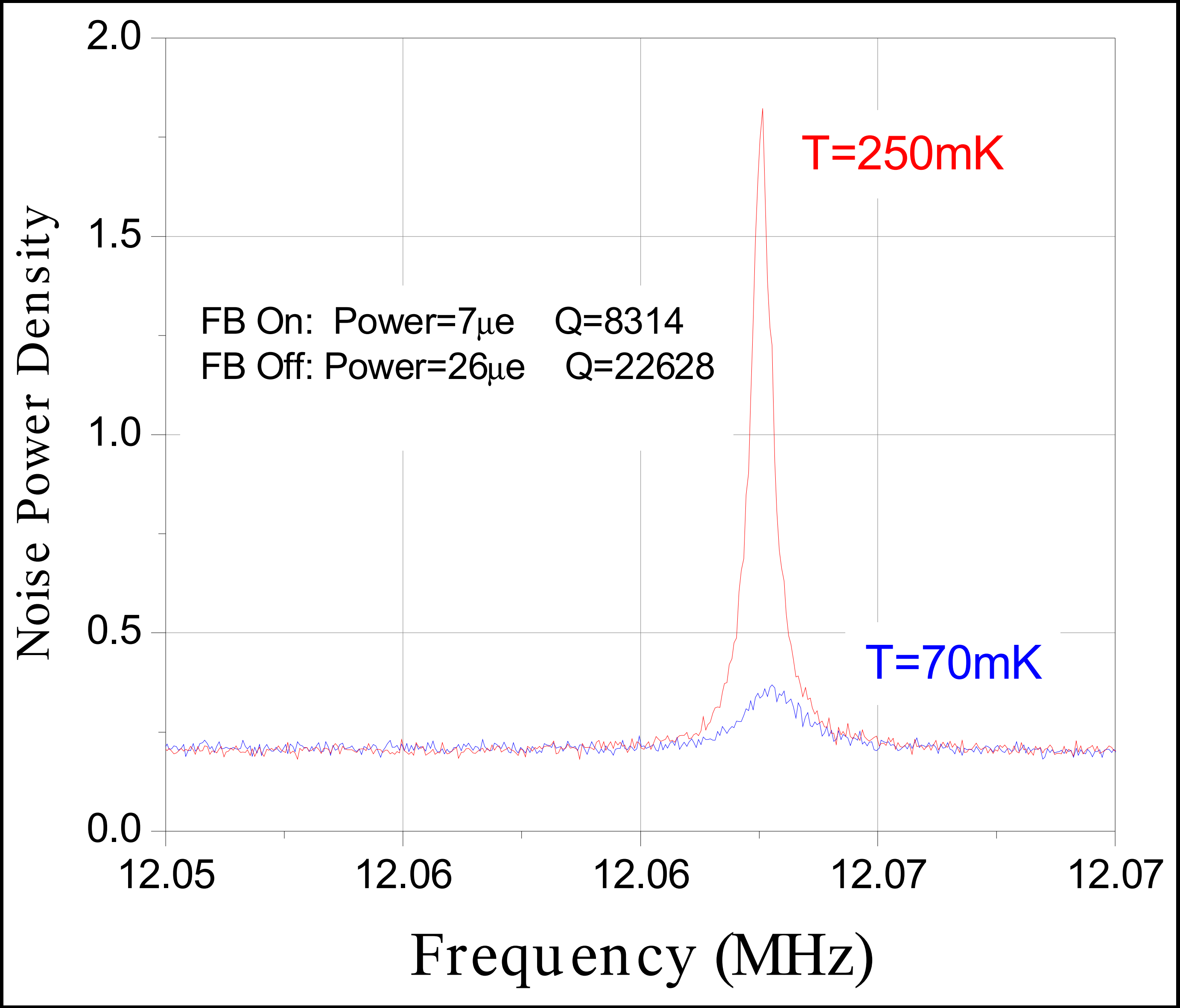
*"Feedback Cooling of a Nanomechanical Resonator" Jacobs, Hopkins, Habib, and Schwab, PRB **68,** 235328 (2004).*

"Quantum Squeezing through Feedback," Korotkov and Schwab, xxx.lanl.gov

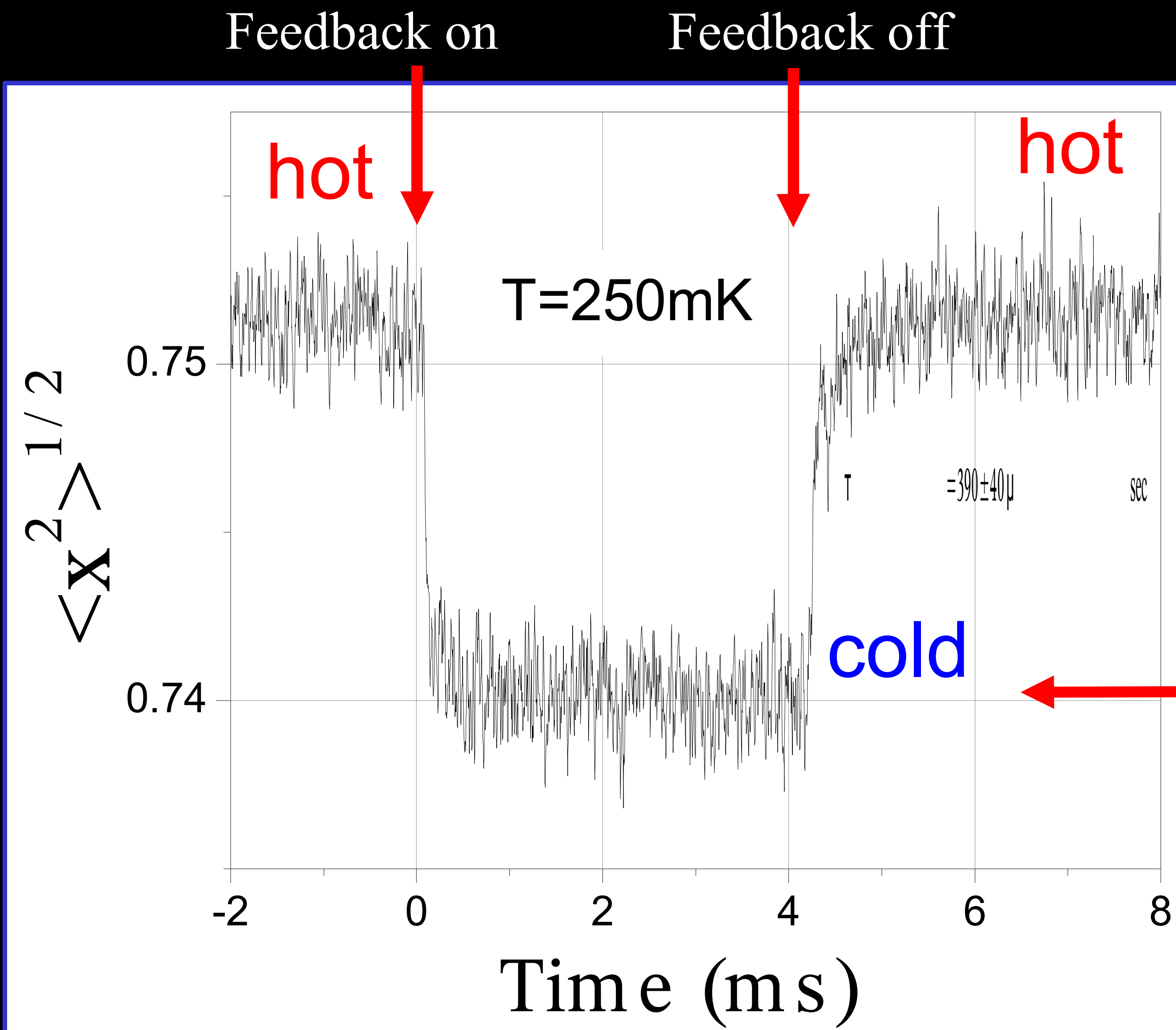


Calculations using a model of continuous quantum measurement, includes all sources of quantum noise (including quantum projection noise and quantum amplifier noise) feedback cooling to N~1-0.1 is possible.

Feedback Cooling-Recent Experiments



Warming and Cooling

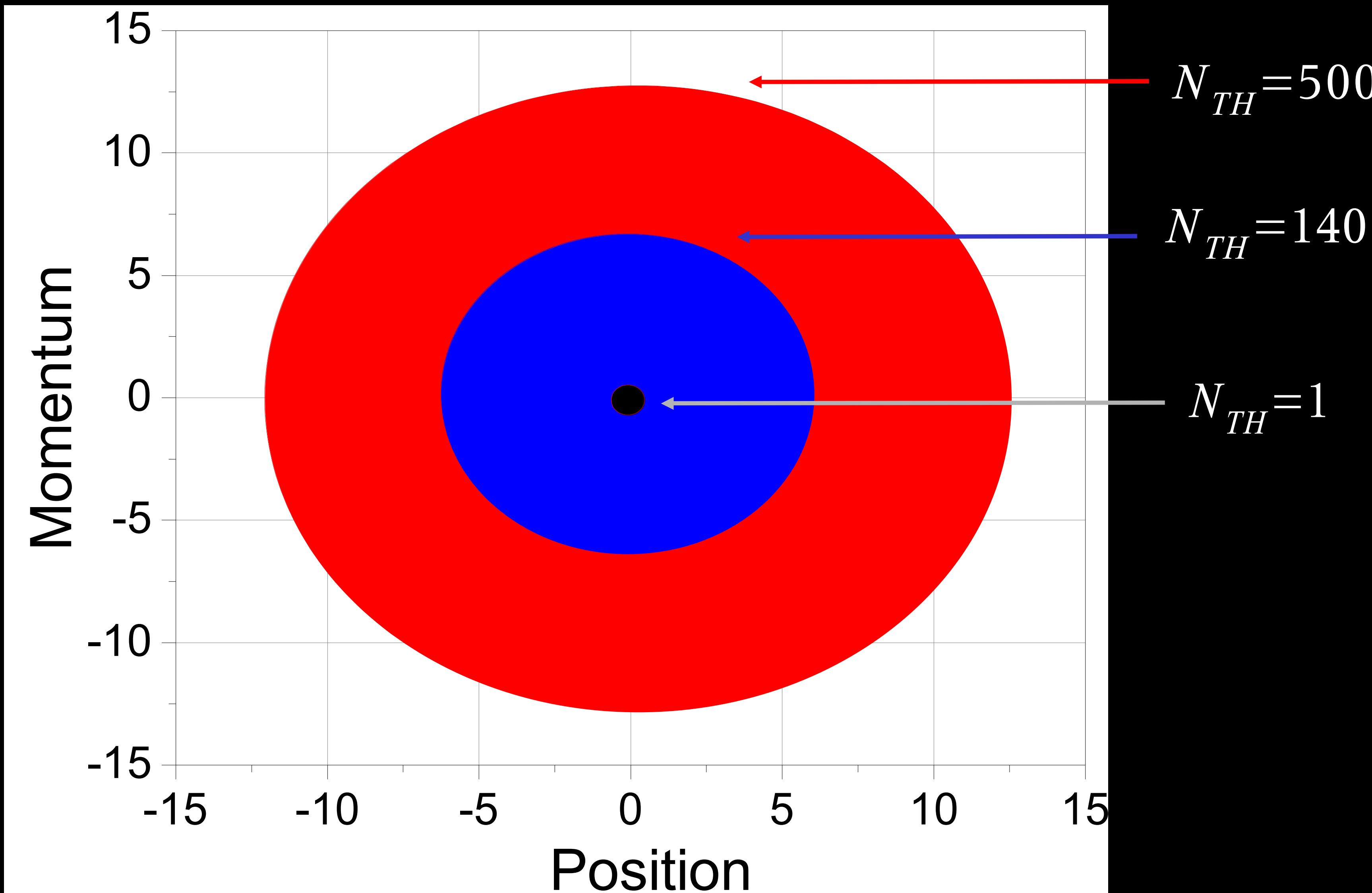


$N_{TH}=500$

$N_{TH}=140$

Thermodynamics of a single degree of freedom

Cooling in Phase Space



Conclusions

- Approached the Uncertainty Principle within a factor ~4.3
  - Back action of SSET at 20 MHz is near ideal
  - Back action at 50 GHz is terrible! (CPB experiments)
- Observed resonator cooled to N ~ 58 $\hbar\omega$
- Starting feedback cooling: thermal relaxation of single mode

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