Quantum Limit of NEMS II

Beyond Linear Detection, Coupling to Qubits

Keith Schwab, Laboratory for Physical Sciences

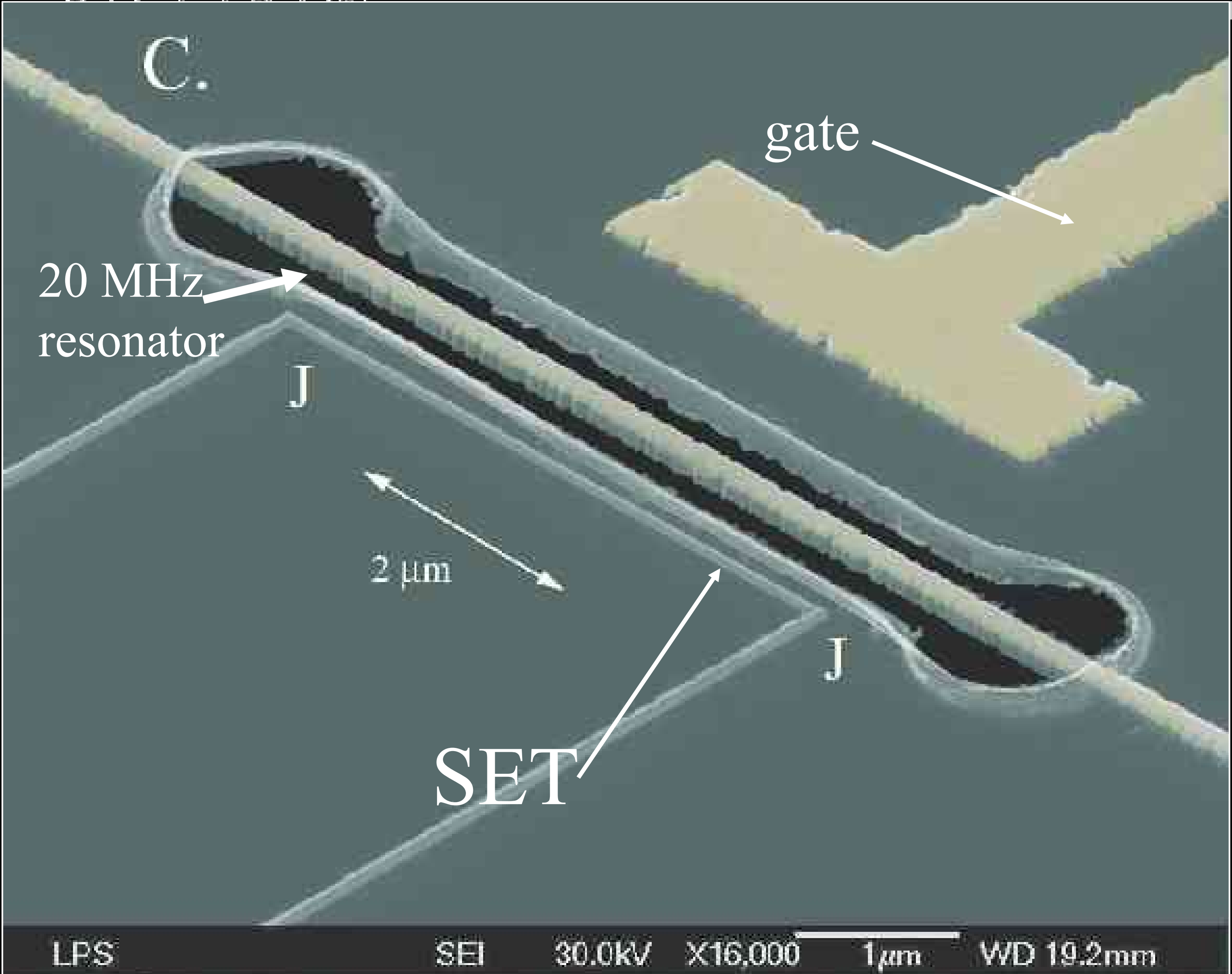
National Security Agency

June 2004

schwab@lps.umd.edu

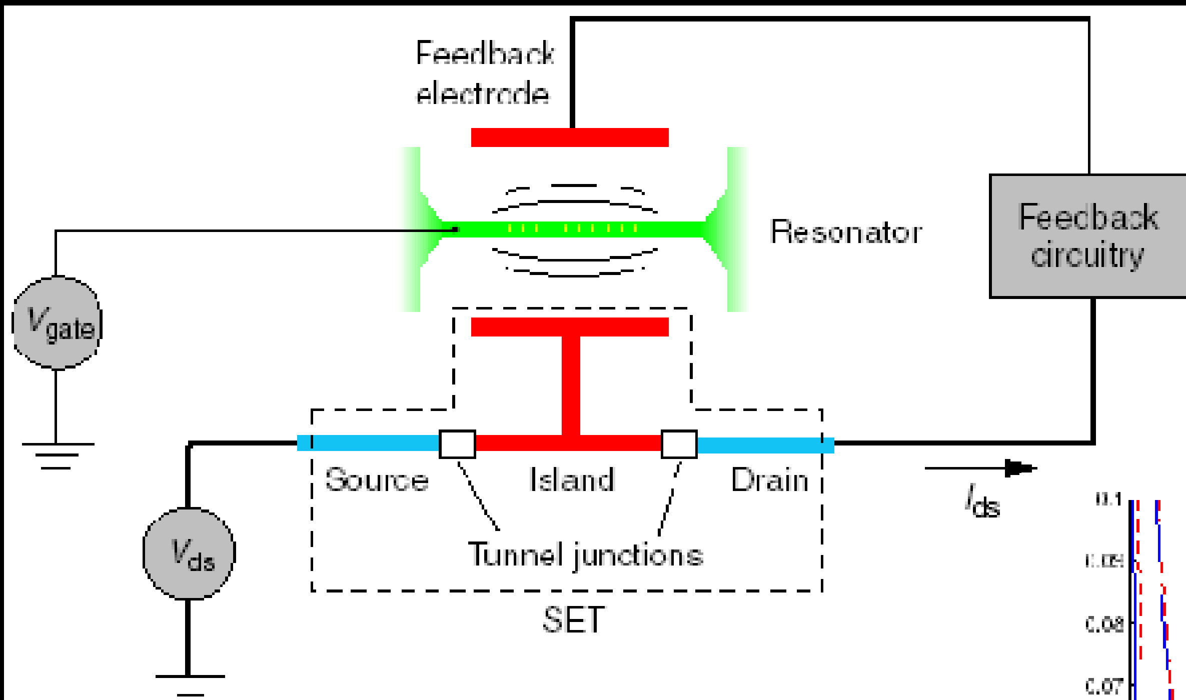
This work is supported entirely by NSA

Recent Devices-RF SET coupled to Nanomechanics



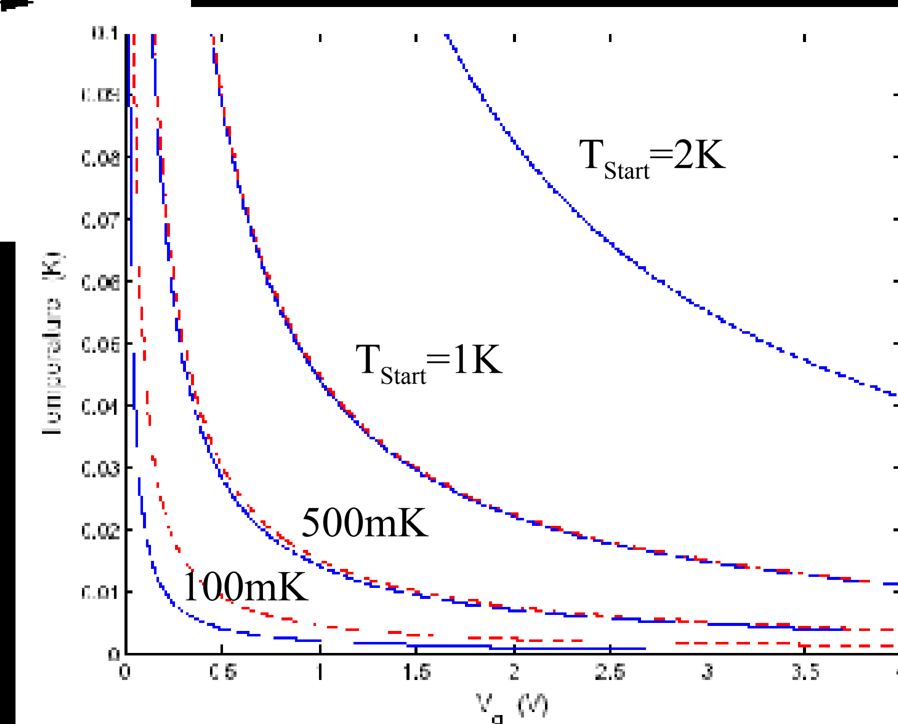
T_N~15mK

Active Cooling to the Ground State



"Feedback Cooling of a Nanomechanical Resonator" Jacobs, Hopkins, Habib, and Schwab, PRB ***68,*** *235328 (2004).*

"Quantum Squeezing through Feedback," Korotkov and Schwab, in preparation

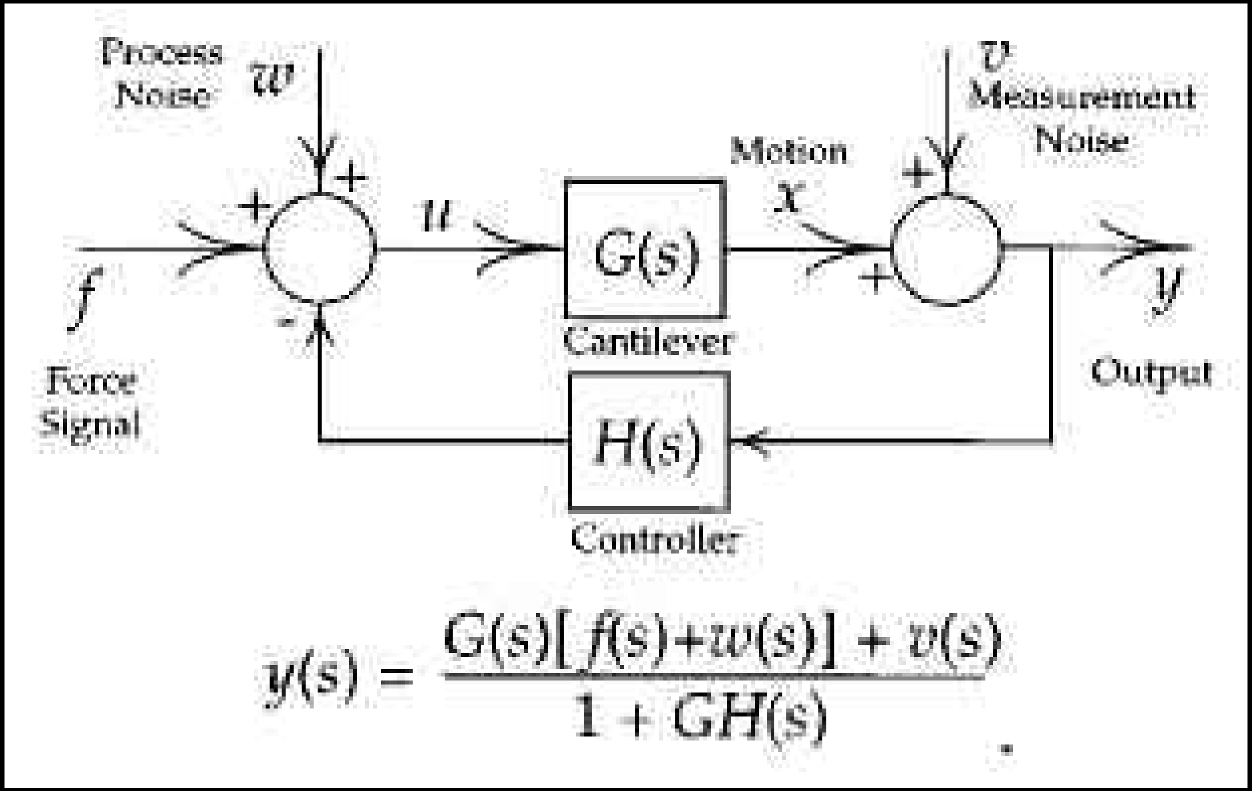


Calculations using a model of continuous quantum measurement, includes all sources of quantum noise (including quantum projection noise and quantum amplifier noise) feedback cooling to N~1-0.1 is possible.

Feedback including QM

Thermal Force Noise
Quantum Back-Action

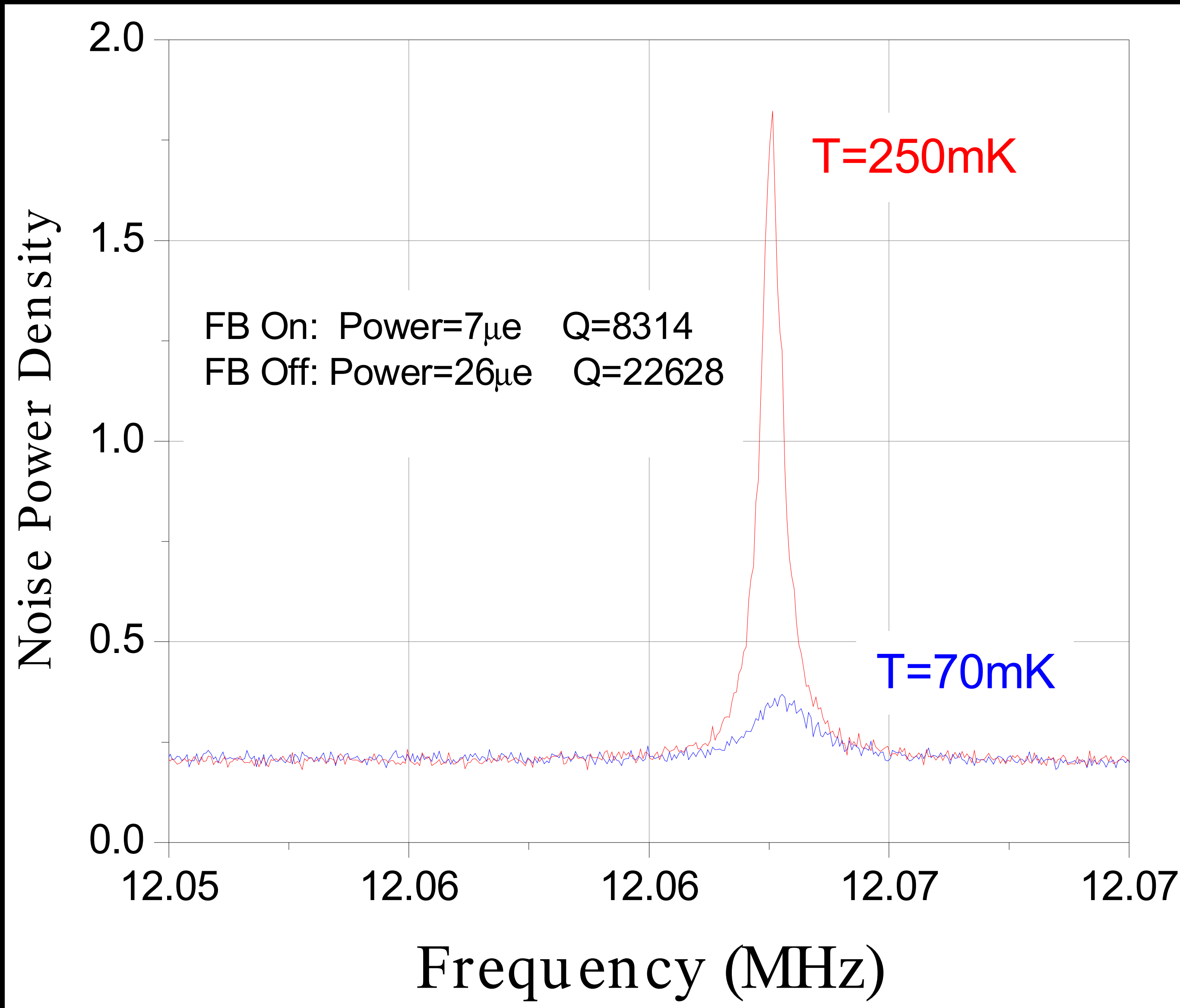
SET Read-out Noise



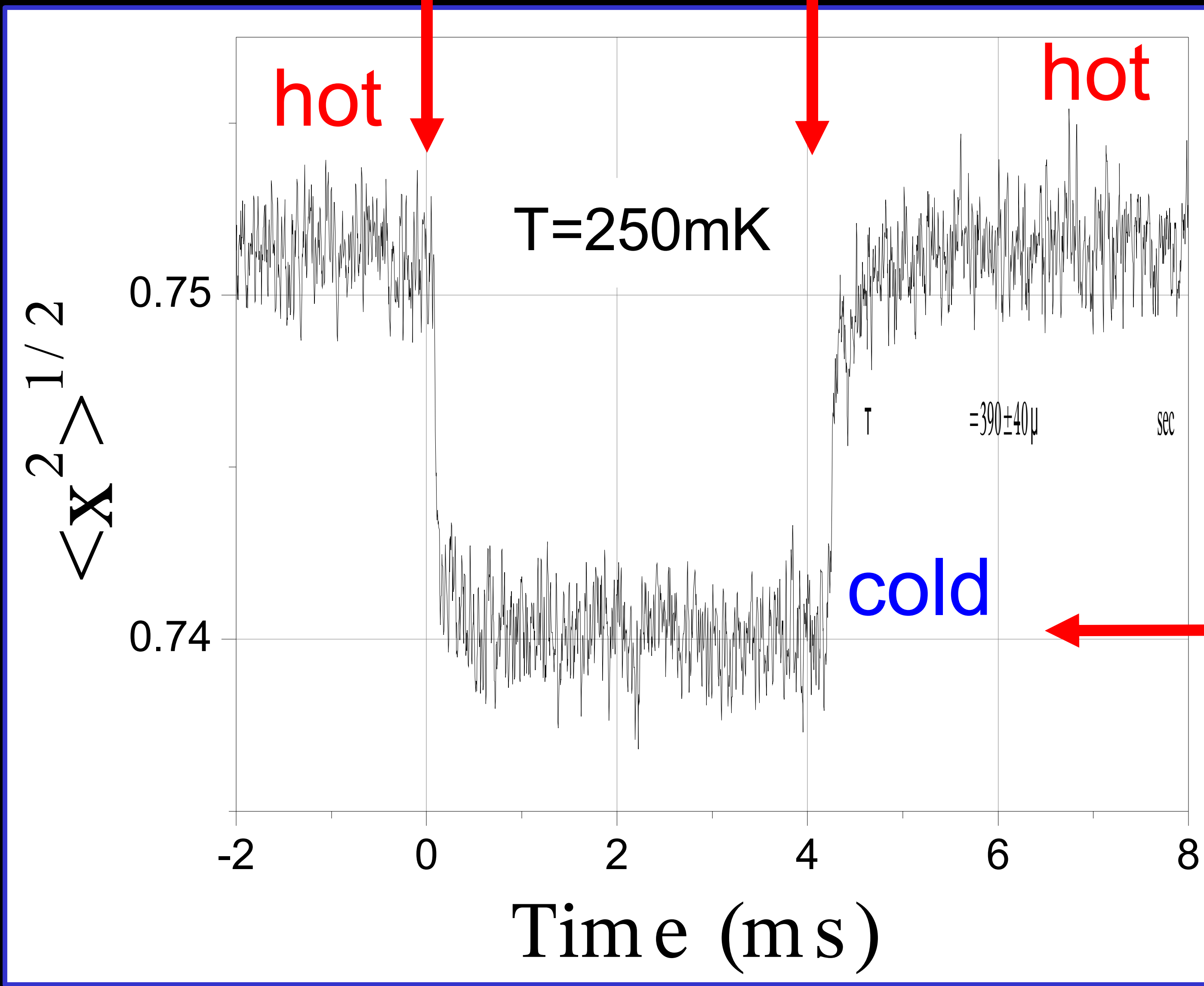
J. Garbini, et al, JAP 1996.

"Feedback Cooling of a Nanomechanical Resonator" Jacobs, Hopkins, Habib, and Schwab, PRB ***68,*** *235328 (2004).*

Feedback Cooling-Recent Experiments



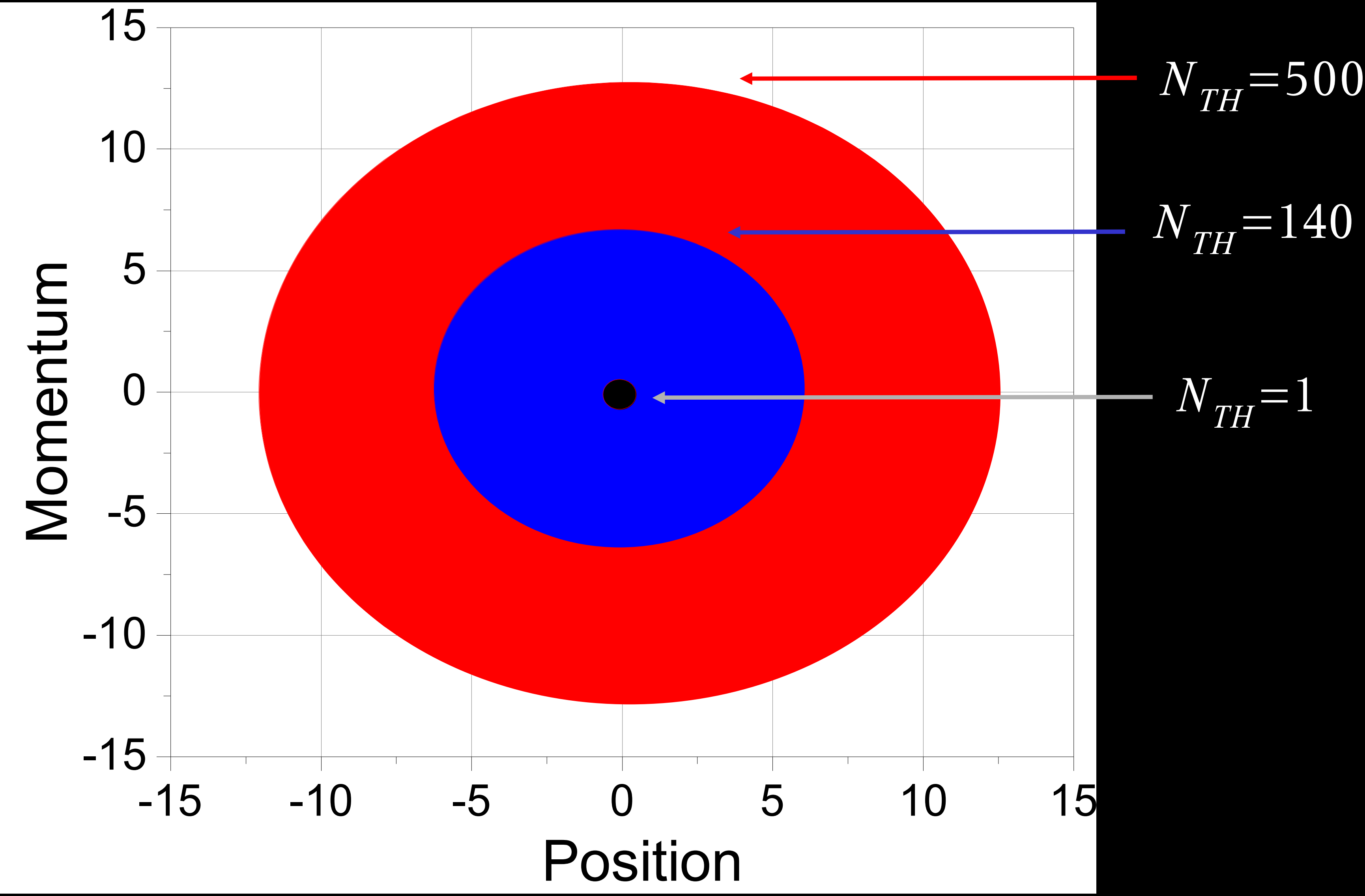
Warming and Cooling



$\tau_m = \frac{Q}{\omega_0} = 300\mu$ sec

Thermodynamics of a single degree of freedom

Cooling in Phase Space

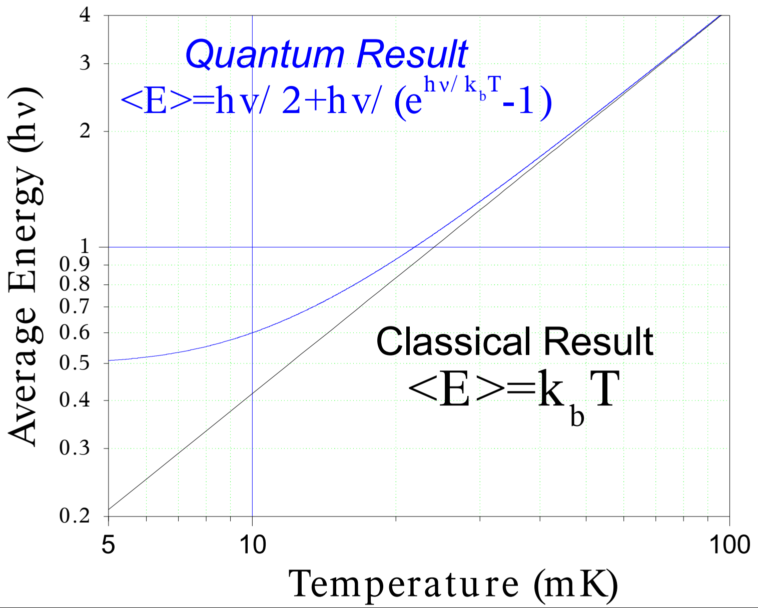


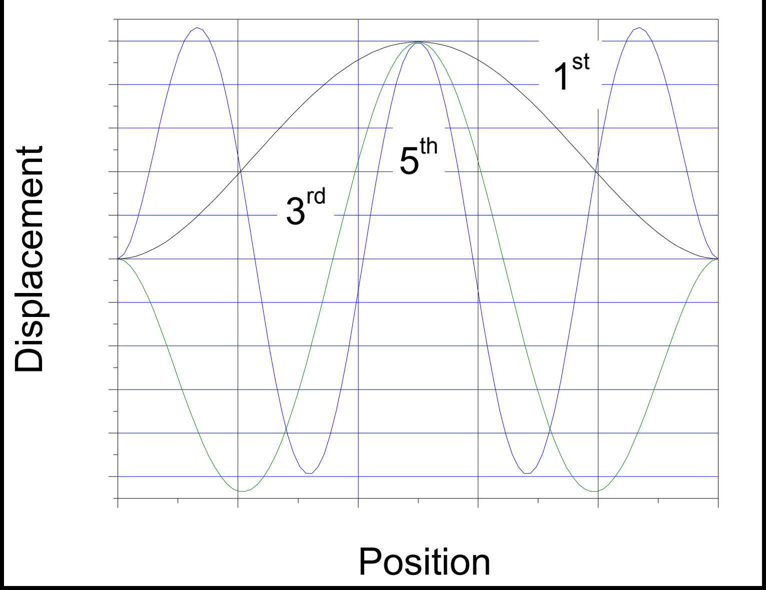
What can we do with QL continuous position measurement?

- <u>rf SSET coupled to a 20 MHz resonator</u>
  - search for higher order modes
  - achieve freeze-out, deviation from classical equipartition

N	*f*	λ
1	20 MHz	8 μm
3	110 MHz	3.4 μm
5	270 MHz	2.1 μm
7	500 MHz	1.6 μm
9	800 MHz	1.3 μm

- <u>rf SSET coupled to a 1 MHz resonator</u>
  - increase coupling by reducing gap, observe back action fluctuations (Armour and Blencowe, Martin and Mozyrsky)
  - drive dc current through resonator observe mech. noise from impact of electrons (Shytov, et al. PRL 2002)– T_N~100mK – 1K
  - explore feed-back cooling(Hopkins, et al. PRL 2004)
    - should be able to cool below T_N< 1 mK
    - expected to be a route to squeezing





Quantum Electro-Mechanics

Quantum Electronics

Nanomechanics

•Single Electron Transistors

•Cooper-Pair Box

•Quantum Dots

•Quantum Point Contacts

•SQUID's

•Single electron spins

…..

$k_B T \sim h\nu$ just a few quanta

$\tau_D > 1/\nu$ long coherence times

QEM

Exploit the quantum electronics to both detect and generate the quantum nature of the mechanical device.

Dynamics follow simple Schrodinger Evolution

Incoherent Single Electronics

SET's

Nanomechanics

Coherent Single Electronics

Cooper Pair Box

Quantum Limited position sensitivity: linear detection

Quantum Back-Action

Squeezing

Tunneling Spectroscopy to reveal energy levels

Quant. Limited Feedback

Coherent Dynamics

Quantum Superpositions

Non-Demolition Measurements

Understanding of Decoherence

Possible Test of Quantum Mechanics

…

Quantum phenomena of the:

***Zeroth kind*:** Wave-like nature becomes apparent in reduced geometries

$$G_{th} = \frac{\pi^2 k_B^2 T}{3h}$$

***First kind*:** Uncertainty Principle-Limited Detection

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

***Second kind*:** Energy Level Quantization

$$E = \hbar\omega\left(n + \frac{1}{2}\right)$$

***Third kind*:** Superpositions and Coherent Evolution

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle + e^{i\frac{\Delta E}{\hbar}t}|\downarrow\rangle\right]$$

***Fourth kind*:** Controlled Entanglement with other quantum systems

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}\left[|\alpha\rangle_1 \otimes |\downarrow\rangle_2 + |\beta\rangle_1 \otimes |\uparrow\rangle_2\right]$$

Some quantum mechanics of simple harmonic oscillators….

Quantum States of an Oscillator

$$H=\frac{1}{2}k\hat{x}^2+\frac{1}{2}\frac{\hat{p}^2}{m}=\hbar\omega_0\left(a^\dagger a+\frac{1}{2}\right)$$

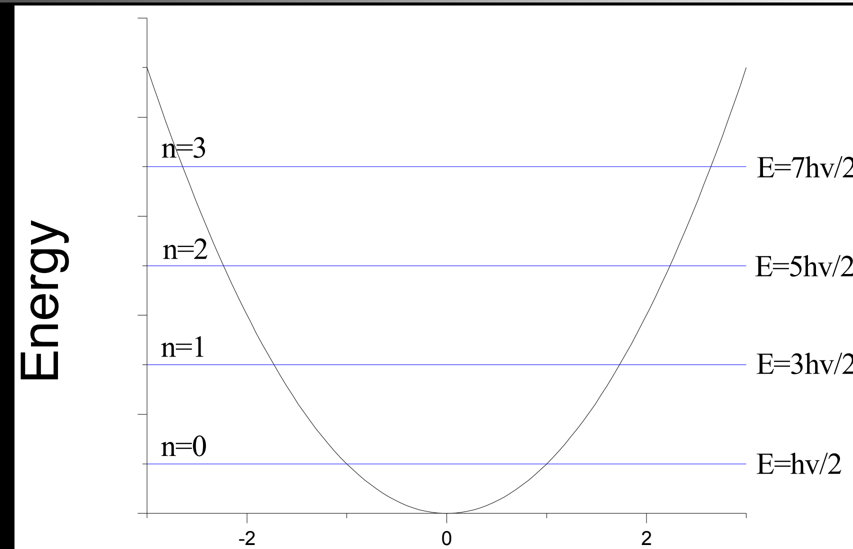
$$\hat{a}^\dagger=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}-i\,\hat{p}/m\omega\right)$$

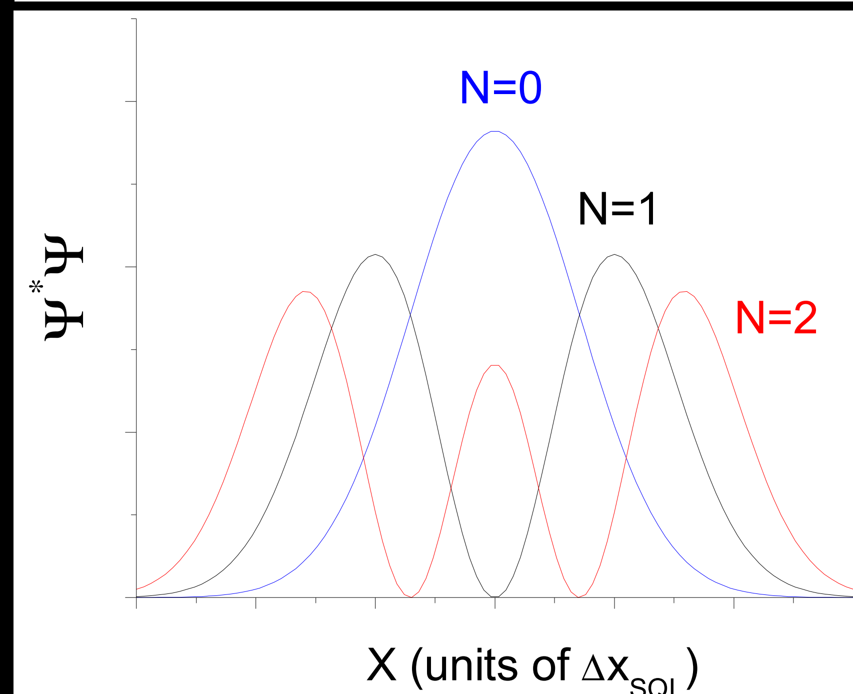
$$\hat{a}=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}+i\,\hat{p}/m\omega\right)$$

Eigenstate: $|N\rangle$

$$E_n=\hbar\omega\left(N+\frac{1}{2}\right)\qquad N=0,1,2,\ldots.$$

“Fock States,” “Number States,” “phonons” equivalent to quantization of the electromagnetic field, photons.





more QM of oscillators…

Raising and Lowering operators:

$$\hat{a}^{\dagger}|N\rangle = \sqrt{N+1}\,|N+1\rangle$$

Creation of a quanta

$$\hat{a}|N\rangle = \sqrt{N}\,|N-1\rangle$$

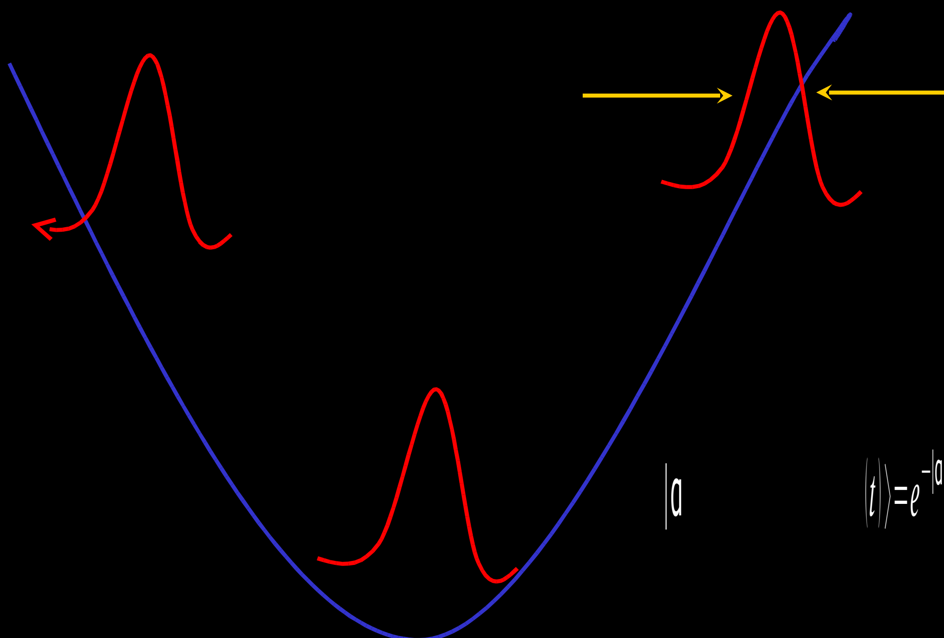
Destruction of a quanta

Superposition States:

$$|\psi(t)\rangle = a_0 e^{i\omega t/2}|0\rangle + a_1 e^{i3\omega t/2}|1\rangle + a_2 e^{i5\omega t/2}|2\rangle + \ldots.$$

$$|\psi(t)\rangle = \sum_{N=0}^{+\infty} a_N e^{i(N+1/2)\omega_0 t}|N\rangle$$

Coherent States



$$\Delta x = \sqrt{\frac{\hbar}{2m\omega_0}}$$

$$|\alpha(t)\rangle = e^{-|\alpha|^2/2} \sum_{N=0}^{+\infty} \frac{\alpha^N}{\sqrt{N!}} e^{-i(N+1/2)\omega_0 t} |N\rangle$$

Coherent States – “classical” quantum states

superposition of number states

wave-packet oscillating in harmonic potential with frequency ω

minimum uncertainty wave packet $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

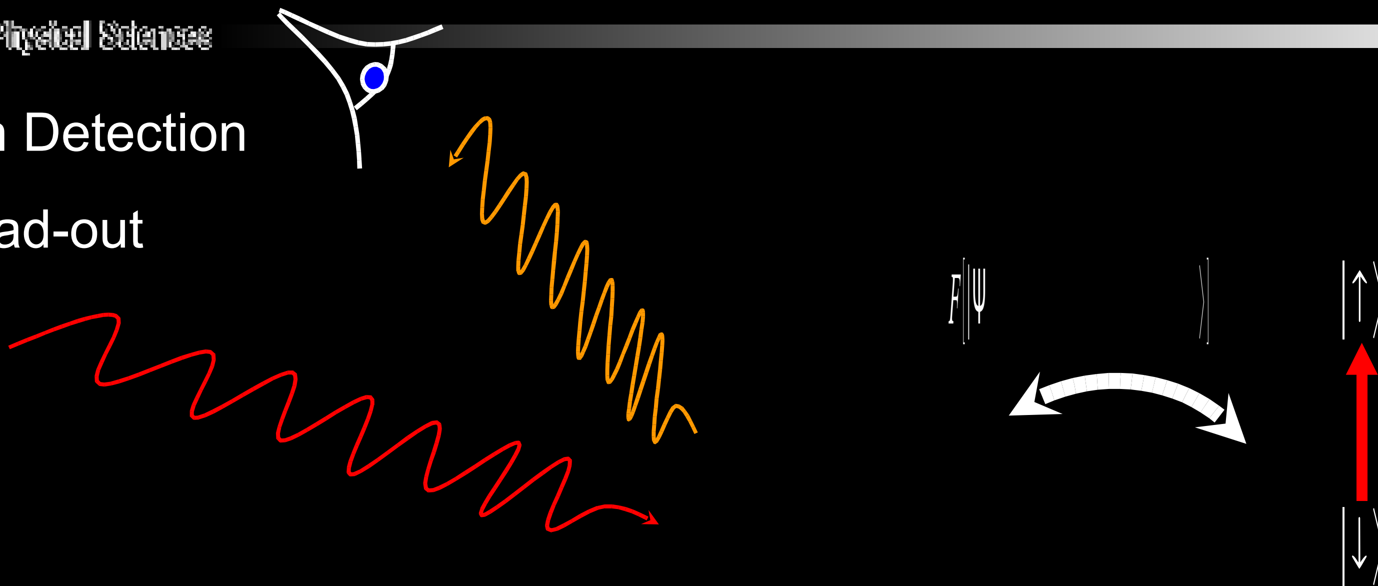
Recipe to make a NEMS device be in two places simultaneously…..

superposition states, entanglements…..

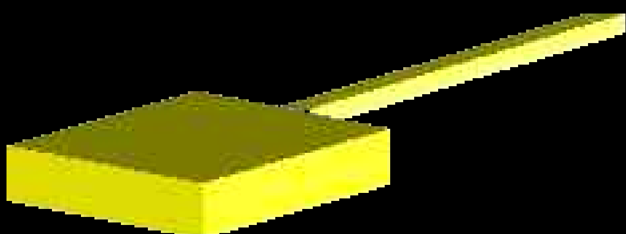
Coupling to Coherent Quantum Devices

Position Detection

Read-out



Quantum Two Level System



10^{-21} Nt ……………….Nuclear Spin

10^{-18} Nt………………. Electron Spin

10^{-13} Nt……. Charge on Cooper-Pair Box

10^{-9} Nt……. Flux in a SQUID ring

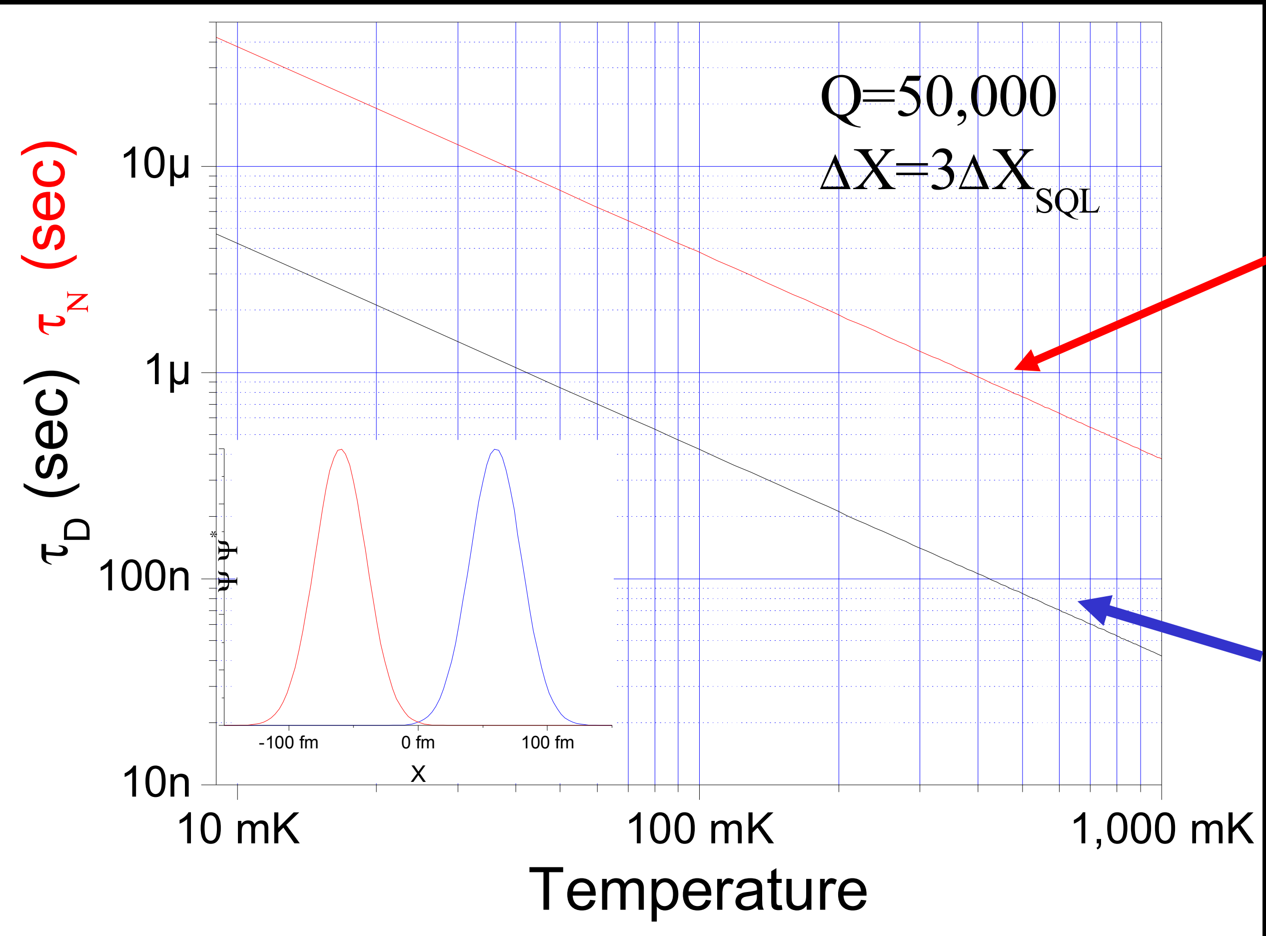
…..

Schrodinger's Cat Situation: Macroscopic state depends on microscopic quantum state

Schrodinger's Whisker

Armour, Blencowe, and Schwab, Phys. Rev. Lett. ***88,*** *148301 (2002).*

Coherence times of the mechanics



Lifetime for number state:

$$\tau_N = Q\frac{\hbar}{k_B T}$$

Decoherence time for superposition of coherent states:

$$\tau_D = \frac{\hbar^2}{2m\gamma_m k_B T (\Delta x)^2} \approx Q\frac{\hbar}{k_B T}\left[\frac{\Delta x_{SQL}}{\Delta x}\right]^2$$

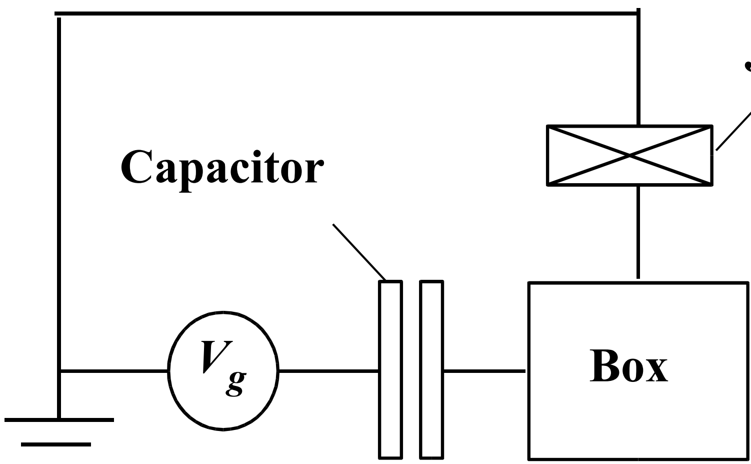
Zurek, Habib, Paz, PRL ***70****, 1187 (1993).*

A little about Qubits……………..

Coherent Single Electronics: Cooper-Pair Box

Ultra-small capacitance ($<10^{-15}$ F) --------- charge eigenstates

Superconducting --------- follows Josephson equations of motion:

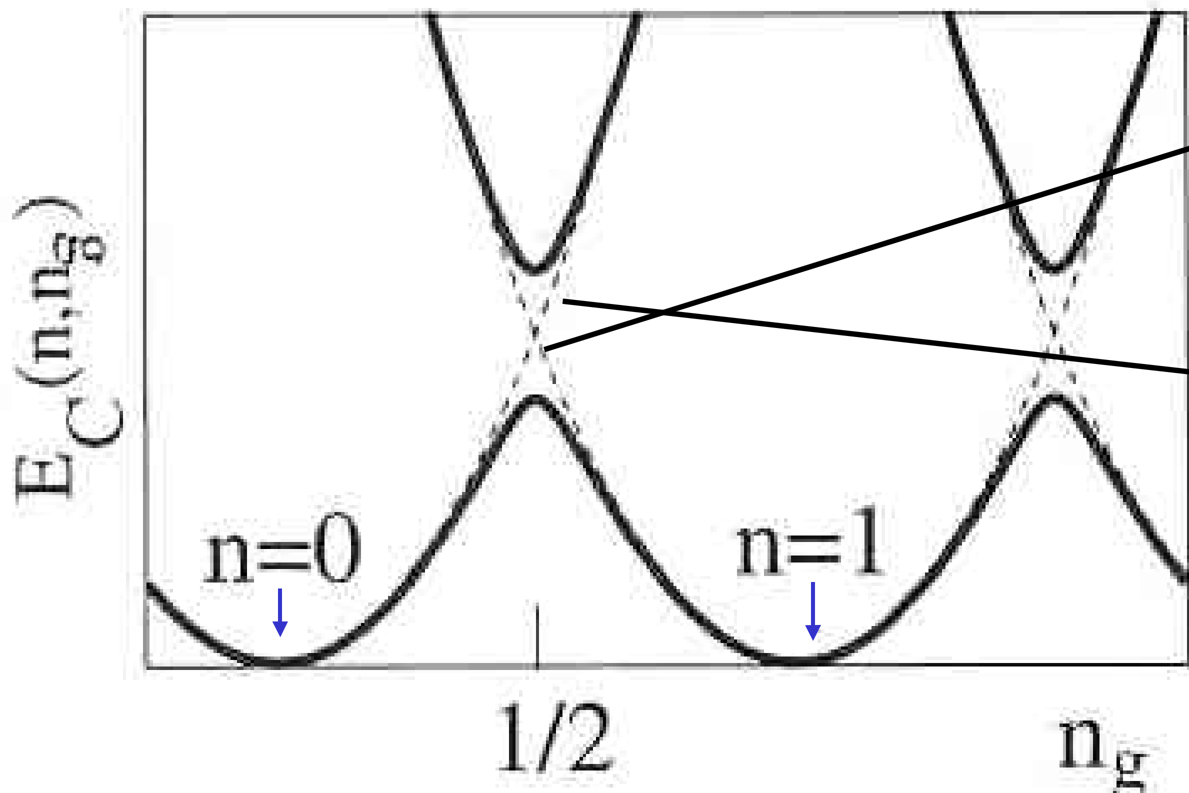


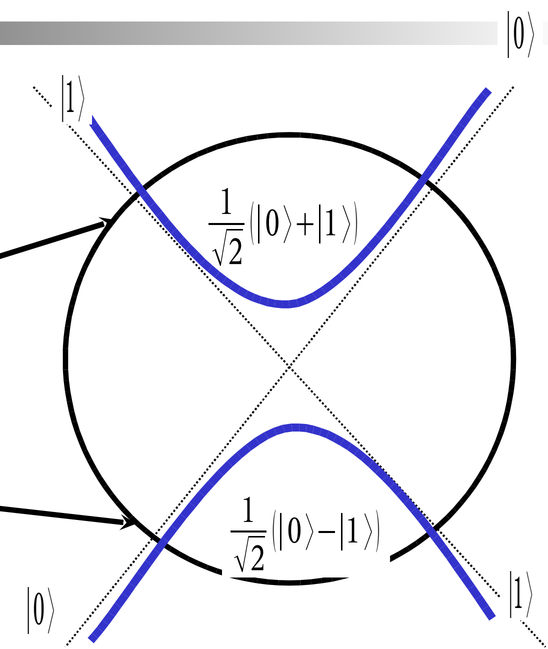
$$H = 4E_C(n - n_g)^2 + E_J \cos\varphi$$

Cooper Pair Binding energy:	Δ=230µV
Charging energy:	$\mathbf{E_c}$ =$e^2/2C$ = 100µV
Josephson energy:	$\mathbf{E_J}$= 50-10µV
Thermal energy:	T = 3µV (30 mK)

$\Delta > E_c > E_J > k_B T$

Energy Spectrum of Cooper-Pair Box





At level crossing, states are superposition of charge states.

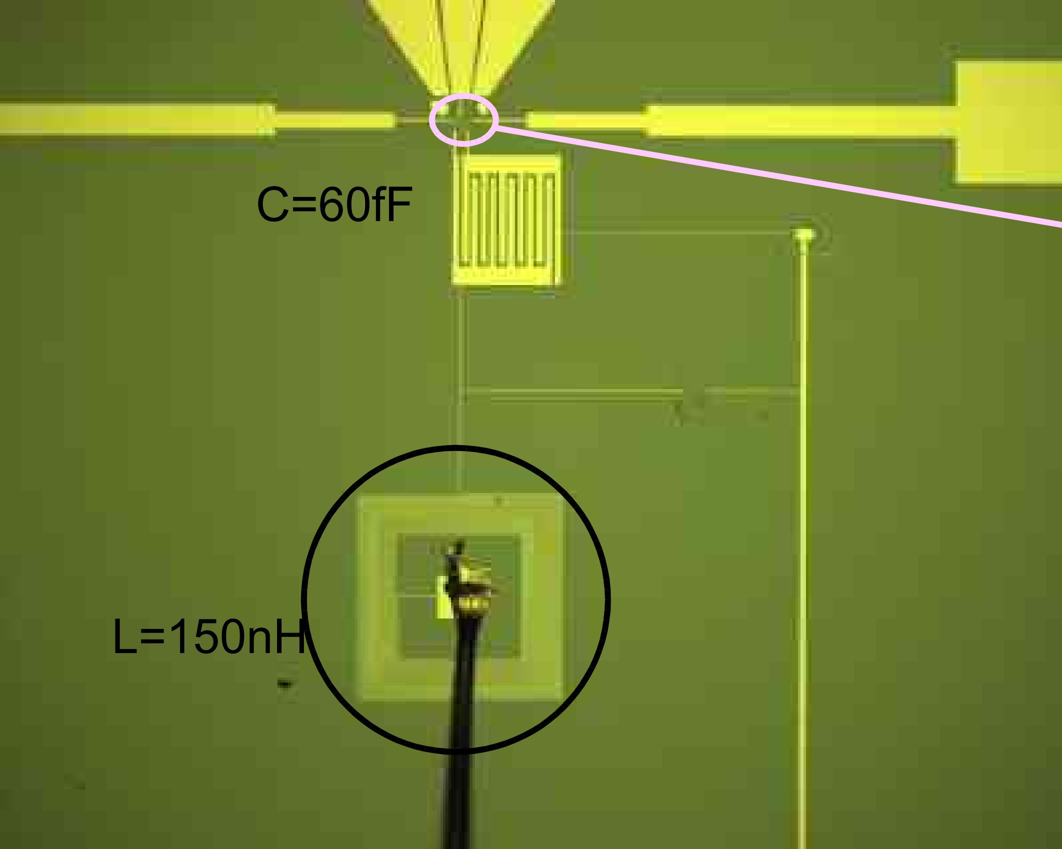
ΔE~50-1 GHz

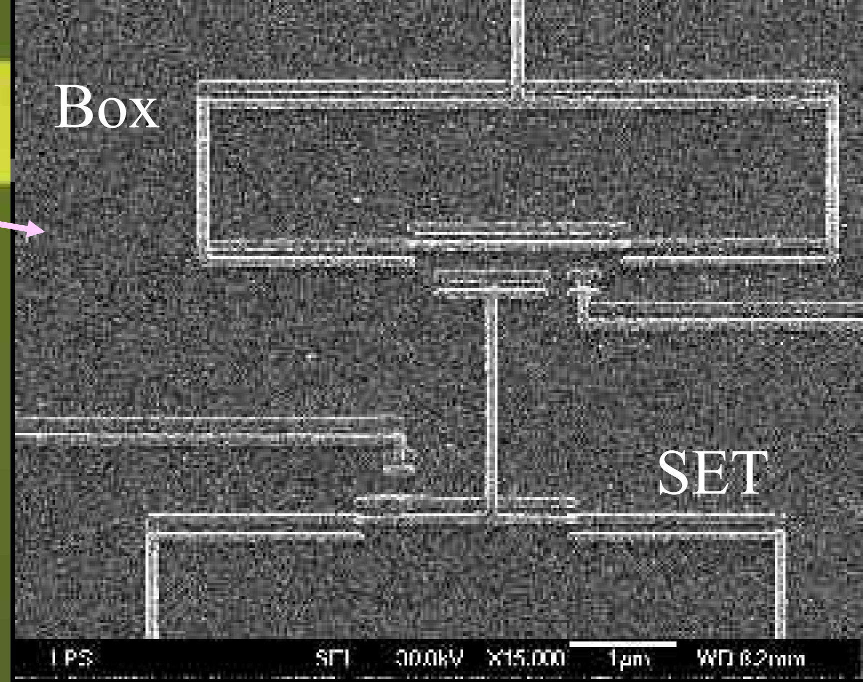
$$H = 4E_C \delta n_g \hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x$$

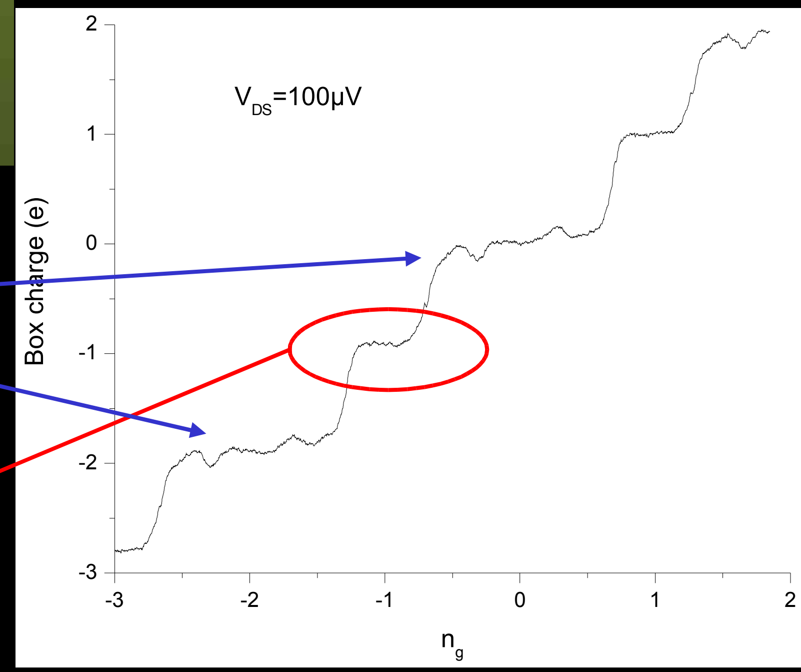
Decoherence time measured to be **t_2~0.5µsec**

And excited state lifetime of **t_1~2µsec**.

Vion, et al, Nature 2002.



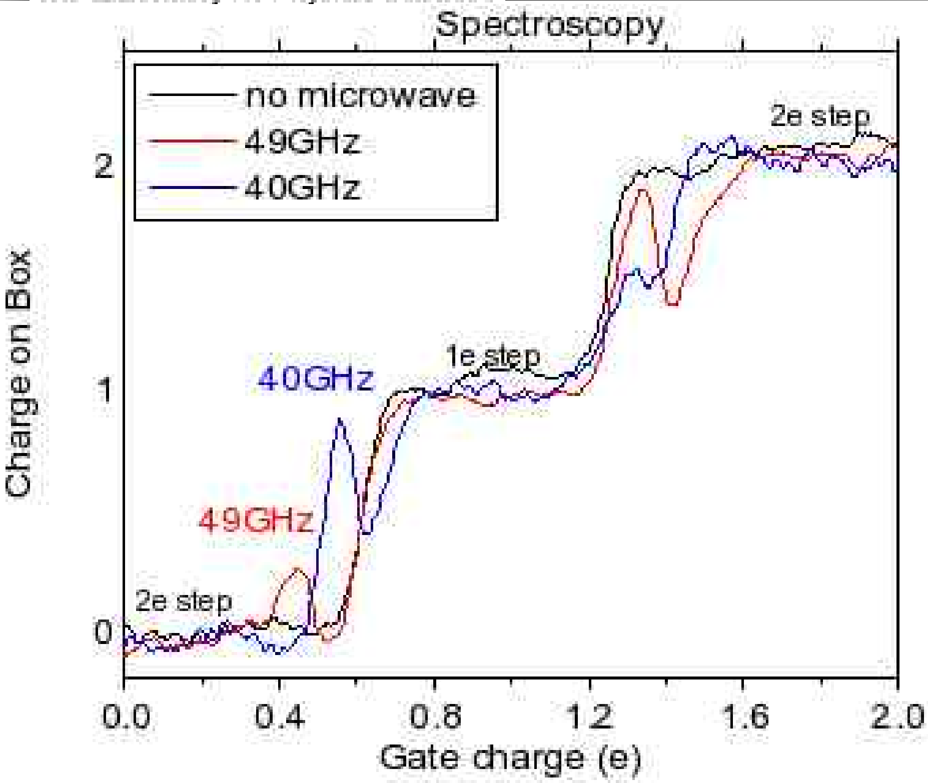


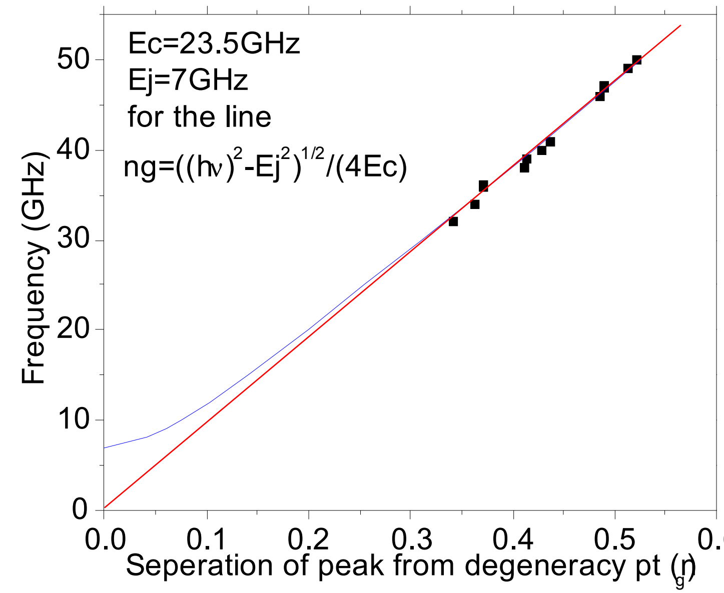


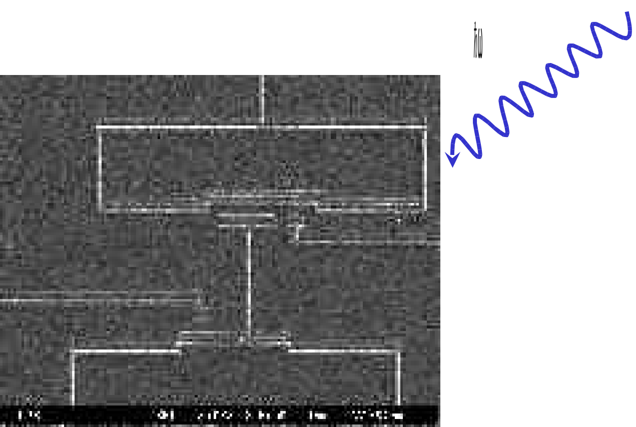
2e-charge states – Good!

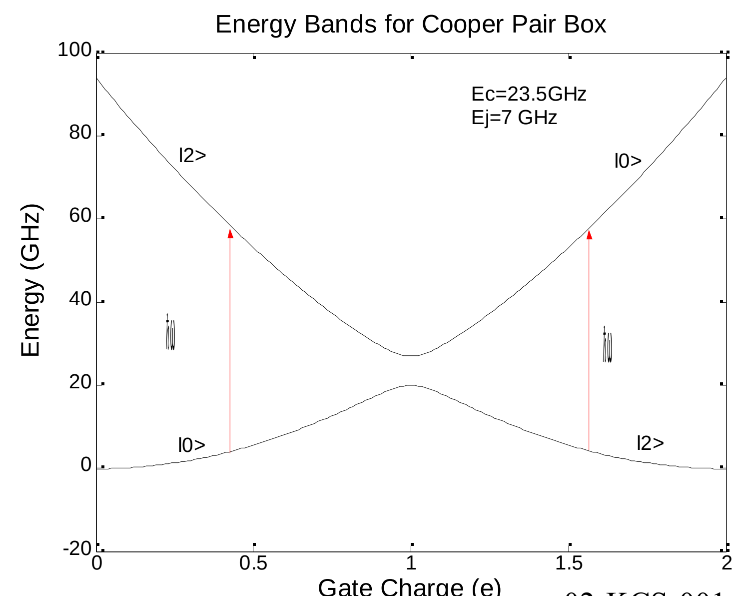
quasi-particle poisoning – Bad!

Spectroscopy of Box



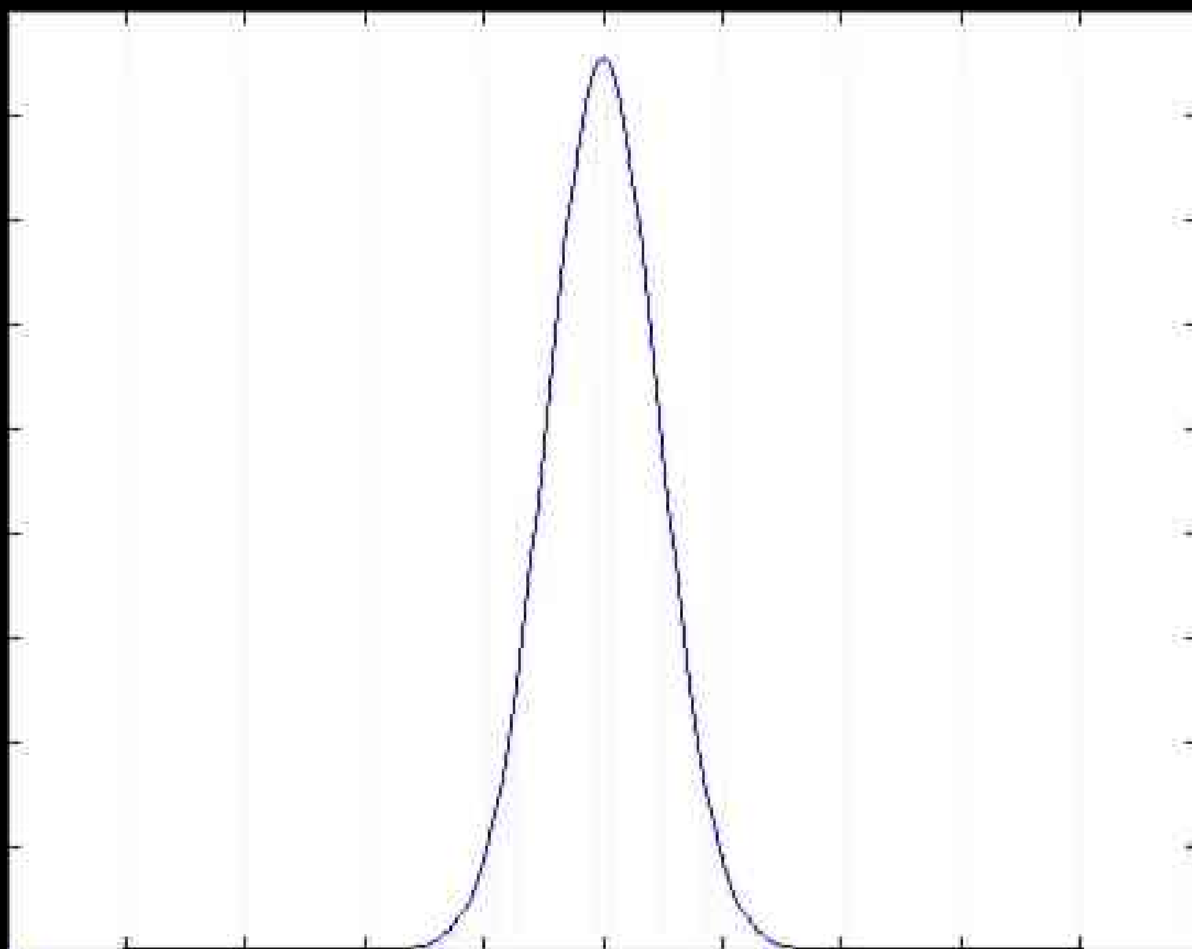






Calculated Dynamics

$$H = 4E_C\delta\hat{n}\hat{\sigma}_z - \frac{1}{2}E_J\hat{\sigma}_x + \hbar\omega\hat{a}^\dagger\hat{a} + \lambda\left(\hat{a}+\hat{a}^\dagger\right)\hat{\sigma}_z$$



Probability Density

1.10^{-13} m

Initial state:

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes|\alpha=0\rangle$$

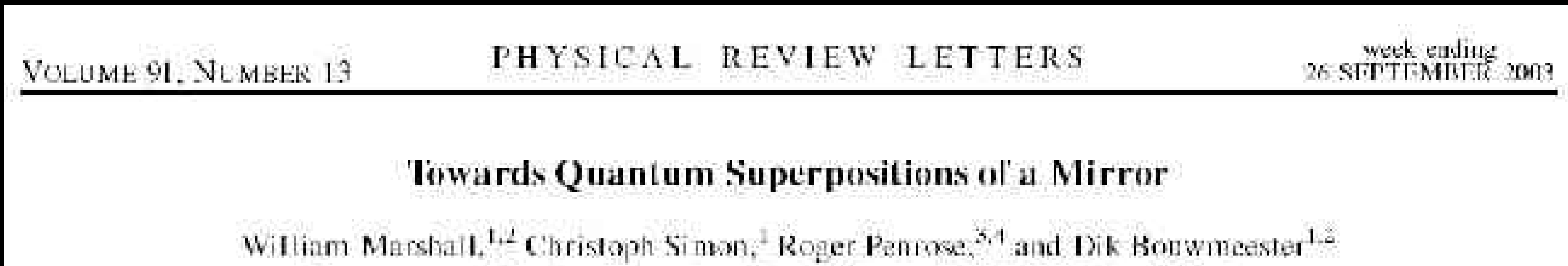
Intermediate state:

$$|0\rangle\otimes|\alpha_0\rangle+|1\rangle\otimes|\alpha_1\rangle$$

After one period:

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes|\alpha=T\rangle$$

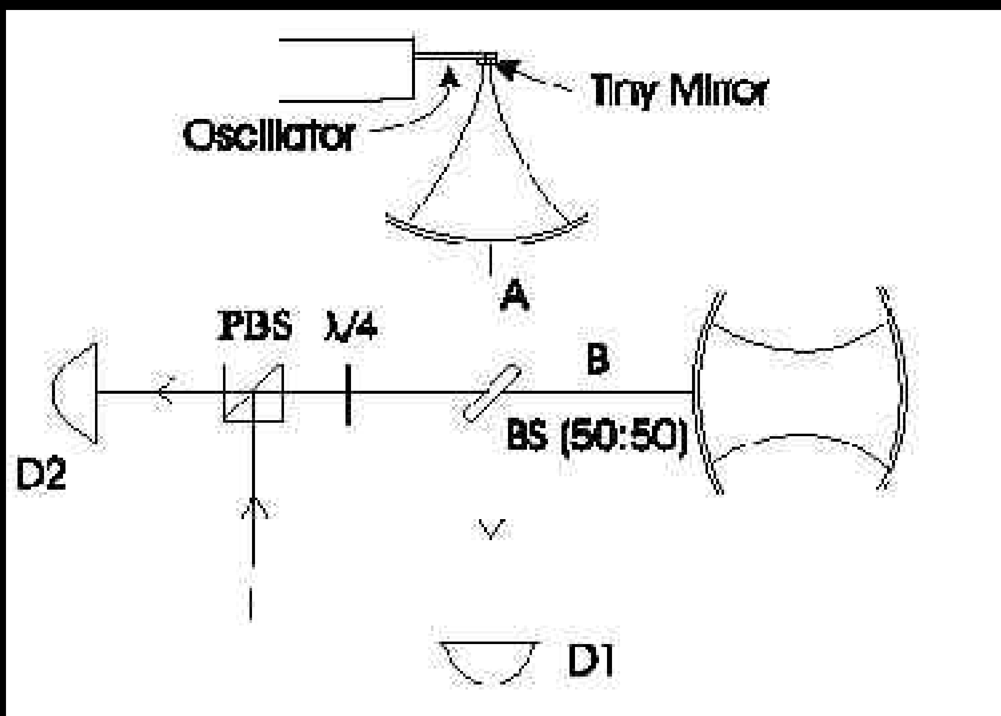
Penrose Proposal



VOLUME 91, NUMBER 13 PHYSICAL REVIEW LETTERS week ending 26 SEPTEMBER 2003

Towards Quantum Superpositions of a Mirror

William Marshall,[1,2] Christoph Simon,[1] Roger Penrose,[3,4] and Dik Bouwmeester[1,2]



Low photon pressure requires very soft cantilever (even after amplify dwell time with cavity)

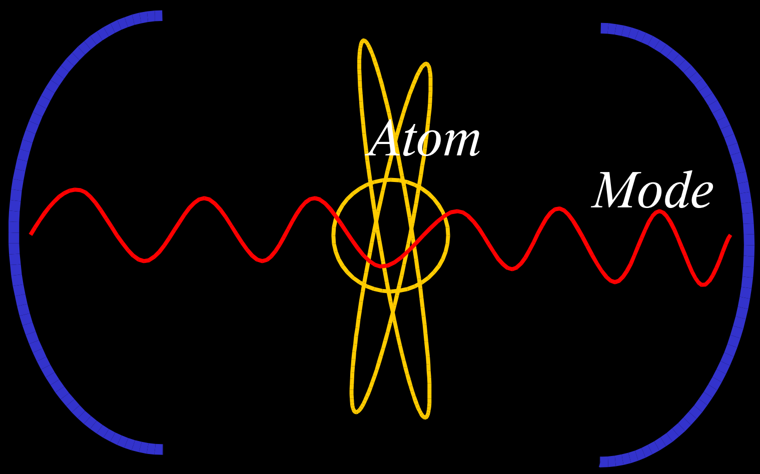
Very soft cantilever has very low frequency ~ 1KHz

Low frequency cantilever has very low freezeout temperature

~ 60μK

Cavity Quantum Electro-Dynamics

$$H_{total} = H_{atom} + H_{field} + H_I$$



$$H_{atom} = \sum_j E_j |j\rangle\langle j| \approx \hbar\omega \quad \cdot\sigma \quad \hat{} \quad z$$

$$H_{field} = \sum_n \hbar\nu \quad {}_n\left(a^+ a + 1/2\right) \approx \hbar\omega \quad {}_0\left(a^+ a + 1/2\right)$$

$$H_I = -e\, r \cdot \vec{E}$$

resonator

Interaction with exchange of quanta

$$H_{total} = \sum_k \hbar\nu_k a_k^+ a_k + \frac{1}{2}\hbar\omega\sigma \quad {}_z + \hbar\sum_k g_k \left(\sigma \quad {}_+ + \sigma \quad {}_-\right)\left(a_k + a_k^+\right)$$

Two state system

Quantum Electro-Mechanics

$$H_{total} = H_{Box} + H_{resonator} + H_I$$

$$H_{box} = \sum_j E_j |j\rangle\langle j|$$

$$¿4E_C \delta n_g \hat{\sigma}_z + \frac{E_J}{2} \hat{\sigma}_x$$

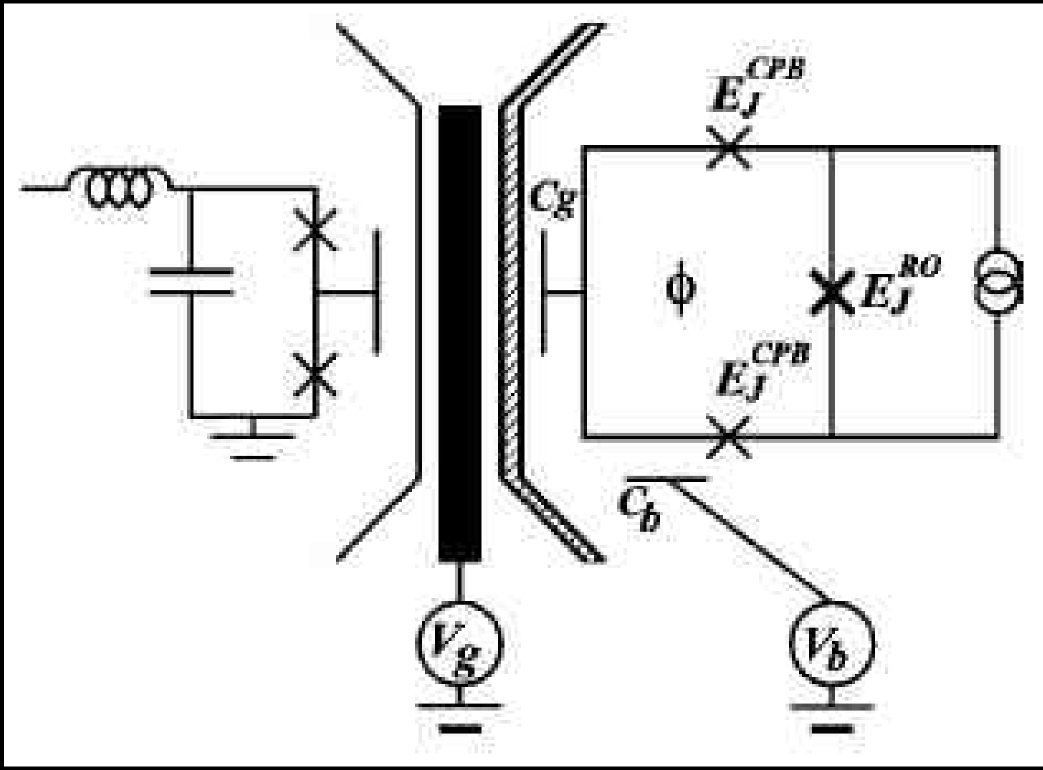
$$H_{resonator} = \sum_n \hbar\omega \left(a_n^+ a_n + 1/2\right)$$

Interaction is through capacitance:

$$H_I = \hat{x}\cdot\vec{F} = \sqrt{\frac{\hbar}{2m\omega_o}}\left(a^+ + a\right)\frac{\partial}{\partial x}\left(\frac{1}{2}CV^2(n)\right)$$

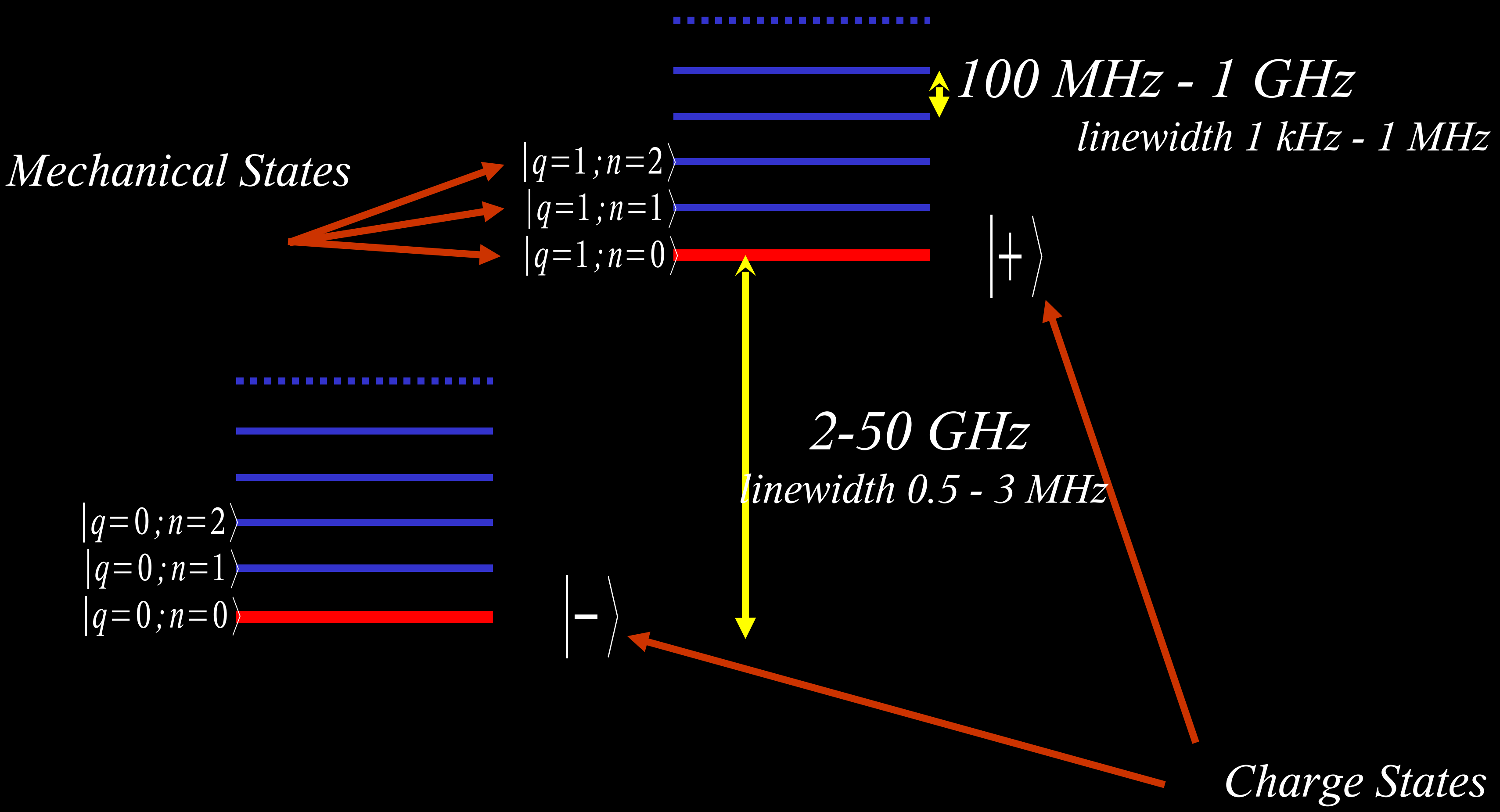
$$¿\lambda \left(a^+ + a\right)\hat{\sigma}_z$$

$$\lambda = 4E_C \frac{C_G V_G}{e}\frac{\Delta x_{ZP}}{d} \approx 5\cdot 10^8 \approx \omega_0$$

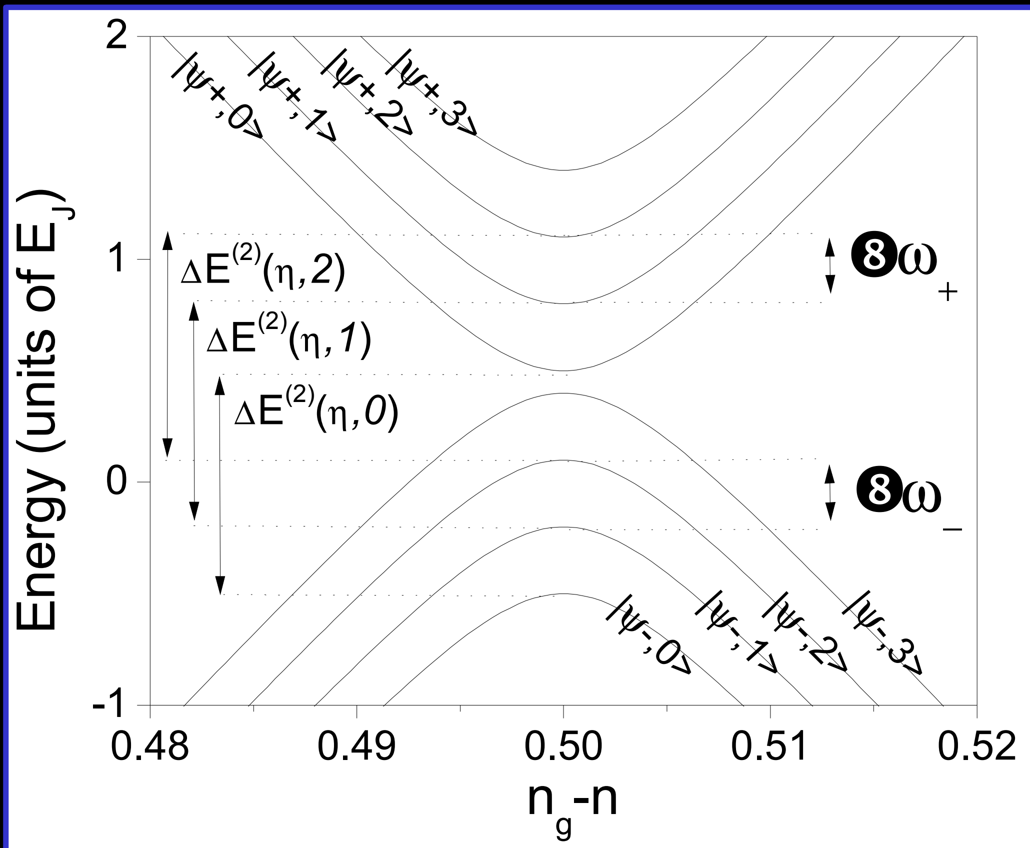


Armour, Blencowe, Schwab, PRL **88**, 148301 (2001).
Armour, Blencowe, Schwab, Physica B **316**, (2002).
Irish and Schwab, PRB **68**, 15531 (2003).

Energy Scales and Linewidths



Mechanical Dressed States



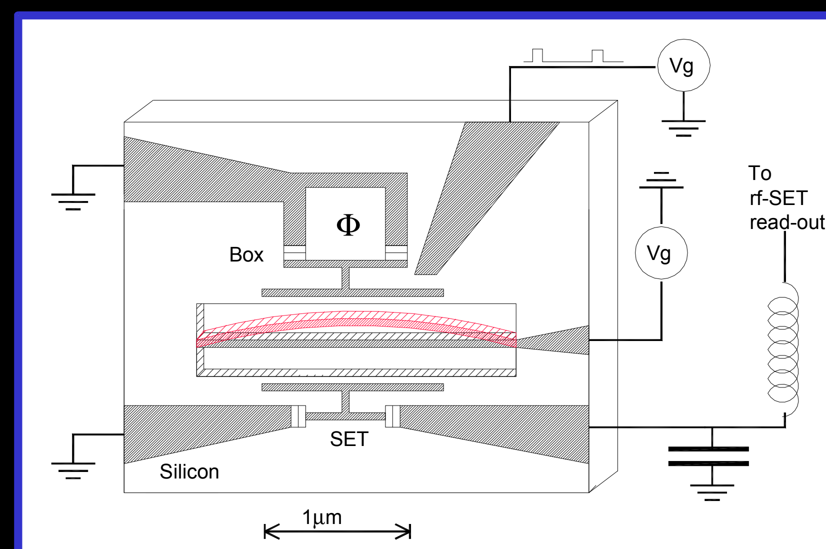
Hamiltonian:

$$H_{System} = H_{CPB} + H_{Resonator} + H_I$$

$$H_{Box} = 4E_C \delta n \cdot \hat{\sigma}_Z - \frac{1}{2} E_J \cdot \hat{\sigma}_X$$

$$H_{Resonator} = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

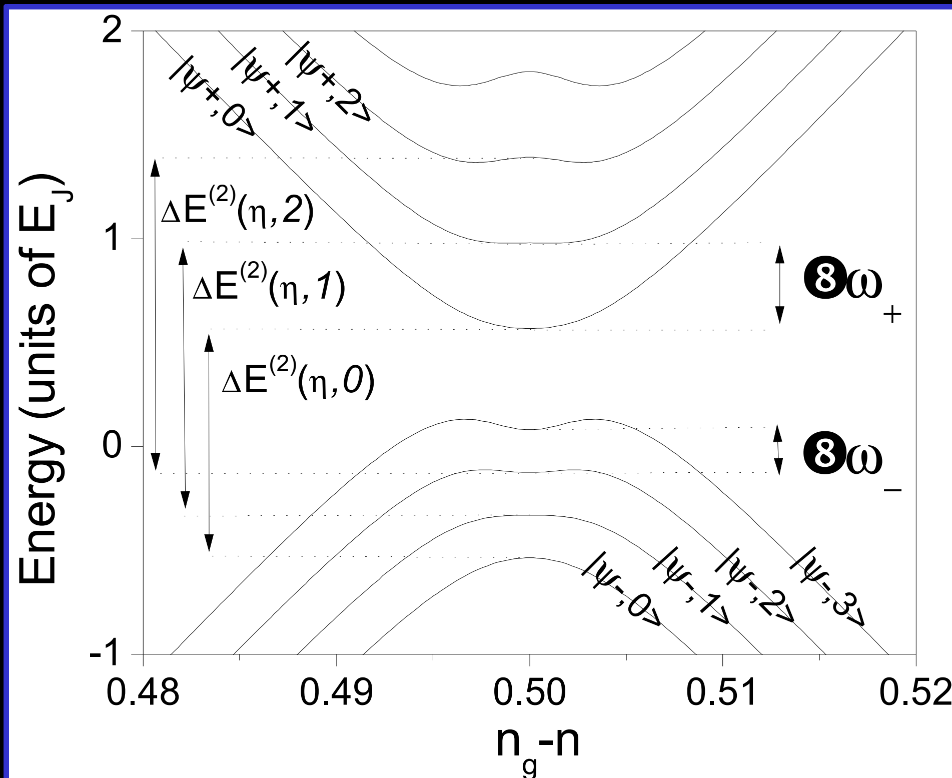
$$H_I = x \cdot F = \lambda\left(\hat{a}^{+} + \hat{a}\right)\hat{\sigma}_Z$$



Ignoring H_I we find the unperturbed energy:

$$\left(H_B + H_R\right)\left|\pm, N\right\rangle = E^{(0)}_{\pm,N}\left|\pm, N\right\rangle = \left(\pm E_B(\eta) + N\hbar\omega_0\right)\left|\pm, N\right\rangle$$

Second Order Shift of Energy Levels



Treat H_I as a perturbation:

$$H_I = x \cdot F = \lambda \left(\hat{a}^{+} + \hat{a}\right)\hat{\sigma}_z$$

$$E^{(2)}_{\pm,N} = E^{(0)}_{\pm,N} + \Delta^{(1)}_{\pm,N} + \Delta^{(2)}_{\pm,N}$$

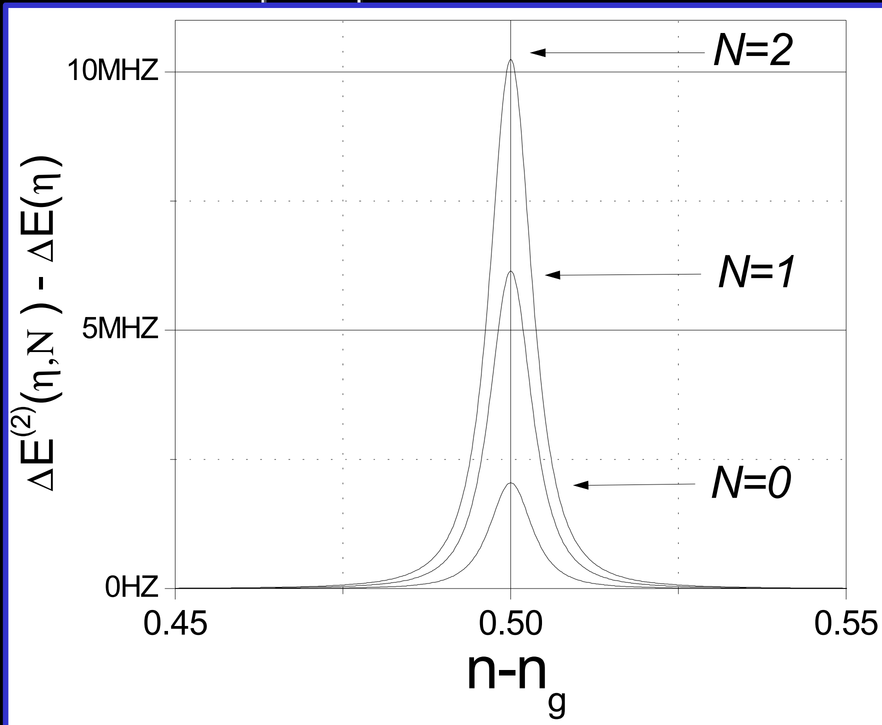
$$\sum_{i} \quad M \quad i = \pm$$

$$\langle N | \left(\hat{a}^{+} + \hat{a}\right) | M \rangle = 0$$

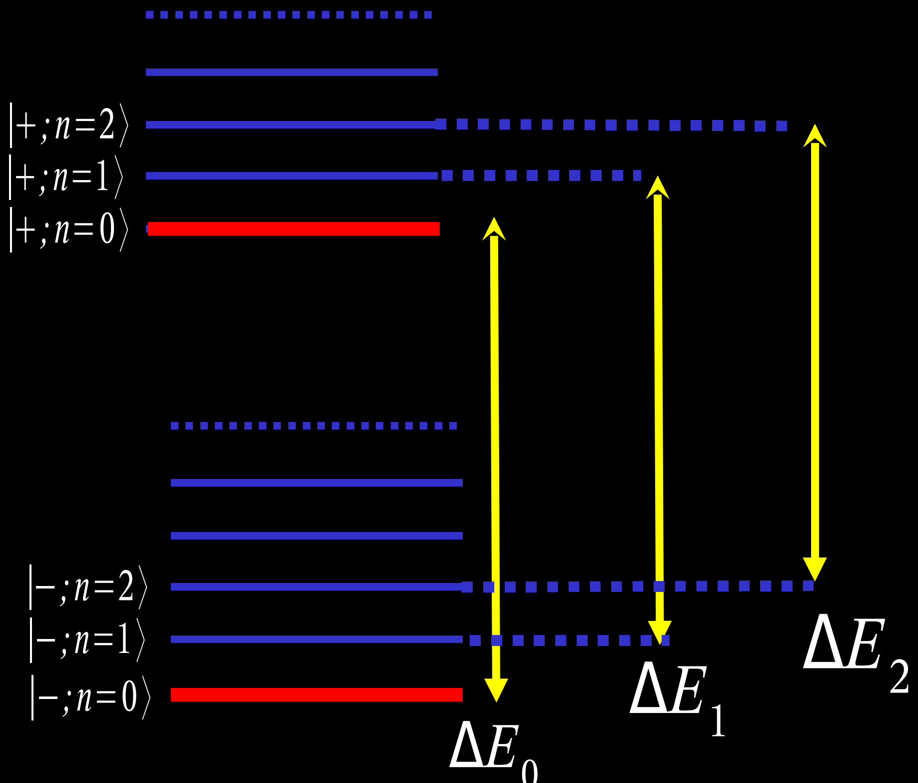
$$\sum \quad M \quad i = \pm$$

"Quantum Measurement with a coupled Nanomechanical Resonator—Cooper Pair Box System," E. Irish and K. Schwab, *Phys. Rev. B* ***68,*** *155311 (2003)*

Shift of the CPB by Resonator Fock States



Mechanical "Lamb shift"



"Mechanical Lamb-shift analogue for the Cooper-pair box," A.D. Armour, M.P. Blencowe, and K.C. Schwab,

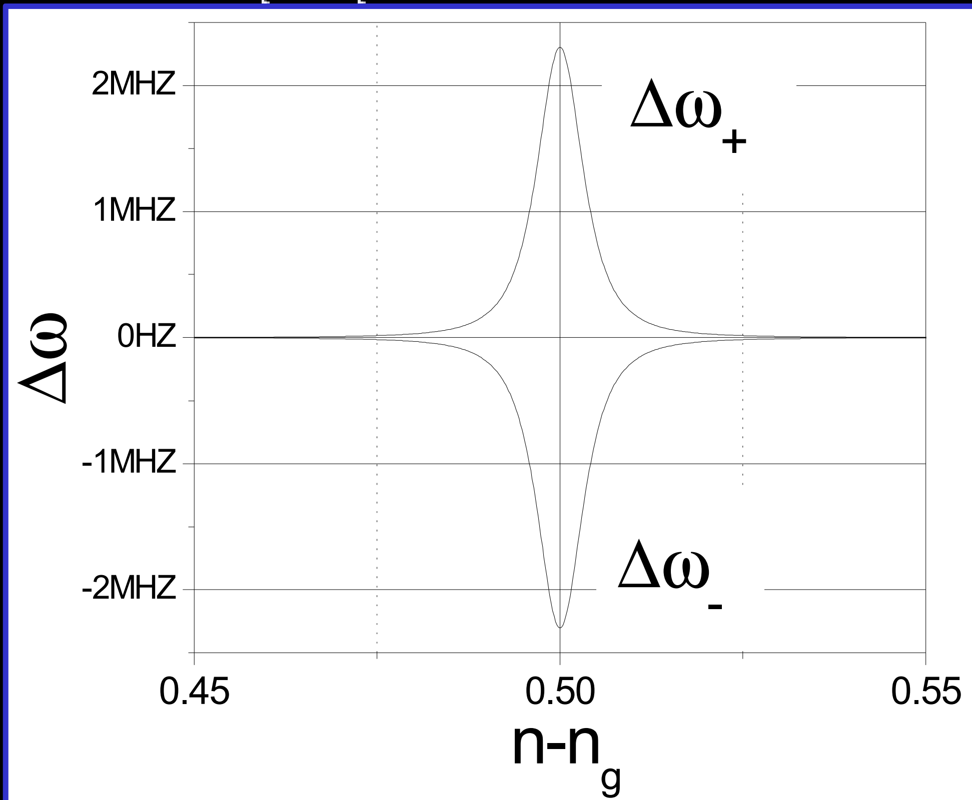
$$\Delta E_N = E_{+,N} - E_{-,N} = 2E_B\left[1+8(2N+1)\sin\eta\,\frac{E_C^2 n_g^2\left(\frac{\Delta x_{zp}}{d}\right)^2}{E_B^2-\left(\frac{\hbar\omega_0}{2}\right)^2}\right]$$

By driving transitions in the Box, one should be able to:

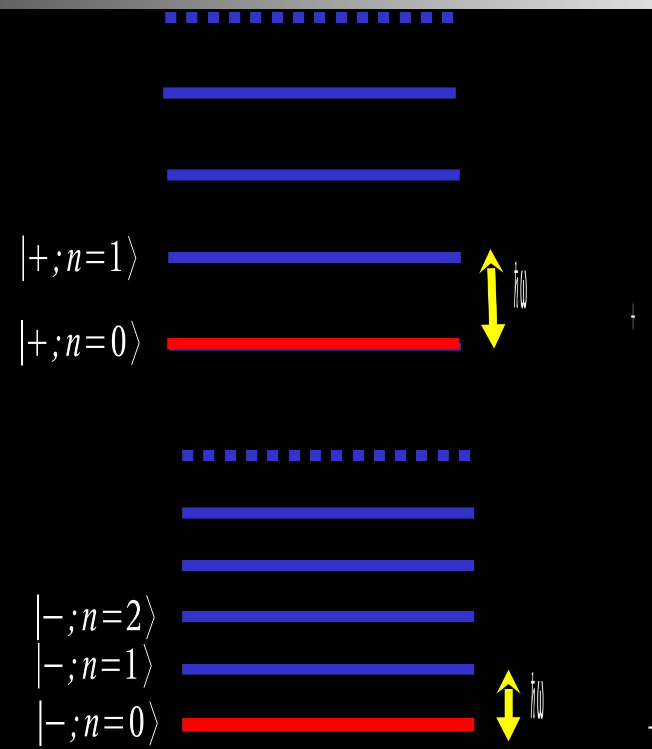
prepare a mechanical number state

perform QND measurement of number using Ramsey interferometry (Vion, 2002)

Shift of the Resonator frequency by the CPB



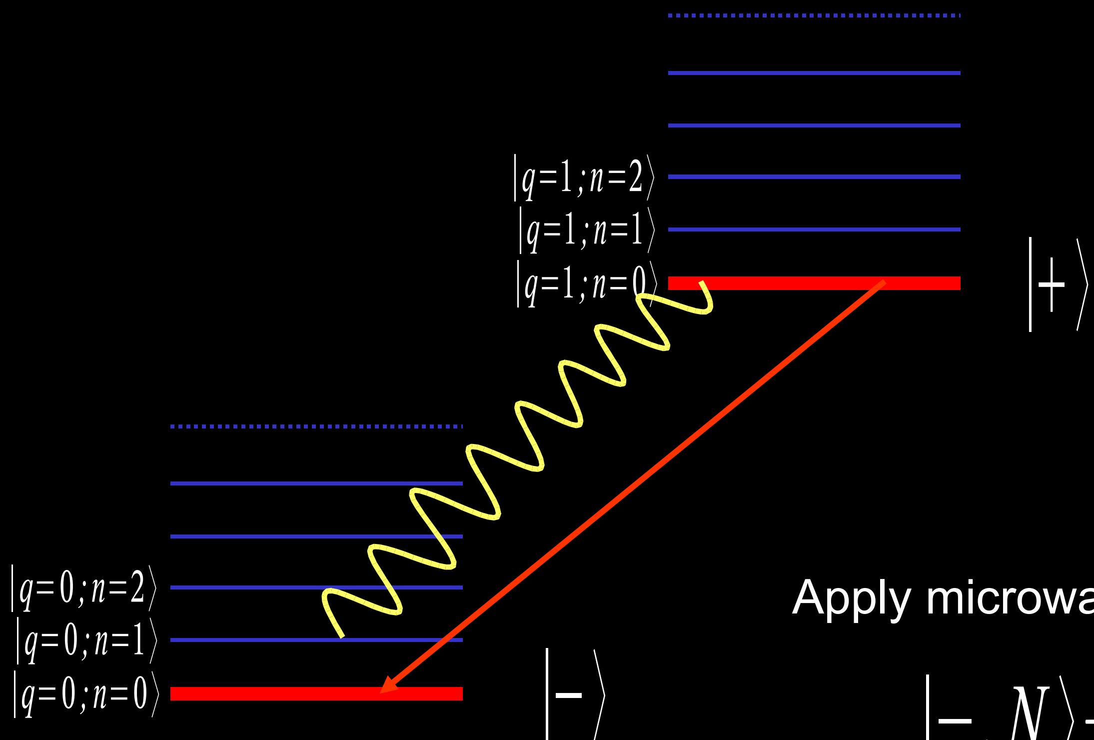
ω_m=300MHz, $\lambda=0.1h\omega_m$, E_J=4μV, E_C=100μV



By measuring the mechanical frequency we can know the state of the phase states of the box.

$$\omega_{\pm} = \frac{E_{\pm,N+1} - E_{\pm,N}}{\hbar} = \omega_0 \pm \left|\frac{\lambda}{\hbar}\right|^2 \frac{E_B}{E_B^2 - \left(\frac{\hbar\omega_0}{2}\right)^2}$$

Mechanical Cooling Through "Laser Cooling" of Qubit?



Apply microwaves to connect:

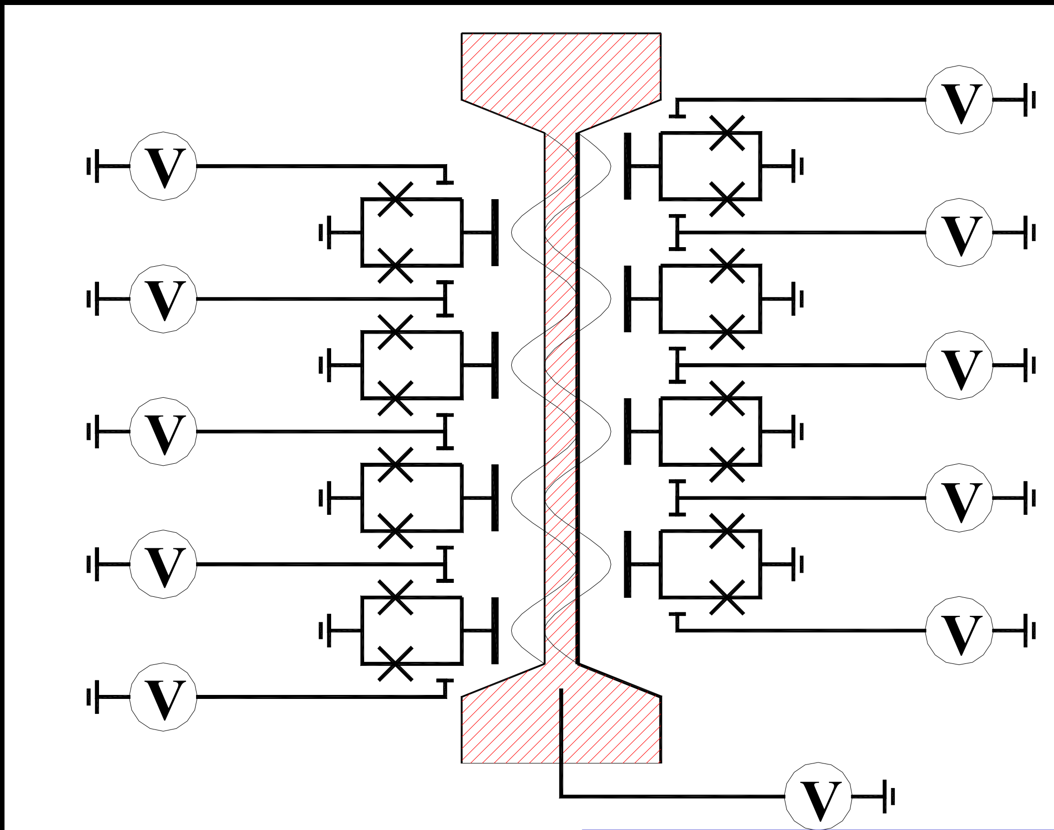
$$|-,N\rangle \rightarrow |+,N-1\rangle$$

Decay of charge state without change in mechanical state:

$$|+,N-1\rangle \rightarrow |-,N-1\rangle$$

"*Ground State Cooling of mechanical resonators,*"
Martin, Shnirman, Tian, and P. Zoller
Phys. Rev. B 69, 125339 (2004)

Nanomechanical Quantum Bus



$$\omega^2 = \left(\frac{Ew^2}{12\rho}\right) k^4$$

For a 1 µm wide Diamond beam (E=1000GPa):

1 GHz λ = 6 µm

10 GHz l = 2 µm

For a 1 µm wide Silicon beam (E=110GPa):

1 GHz l = 3.5 µm

10 GHz l = 1.2 µm

What is the Q of these high

What is the effect of the oth
Possible source of decohere

Advanced materials: Piezo
nanotubes?

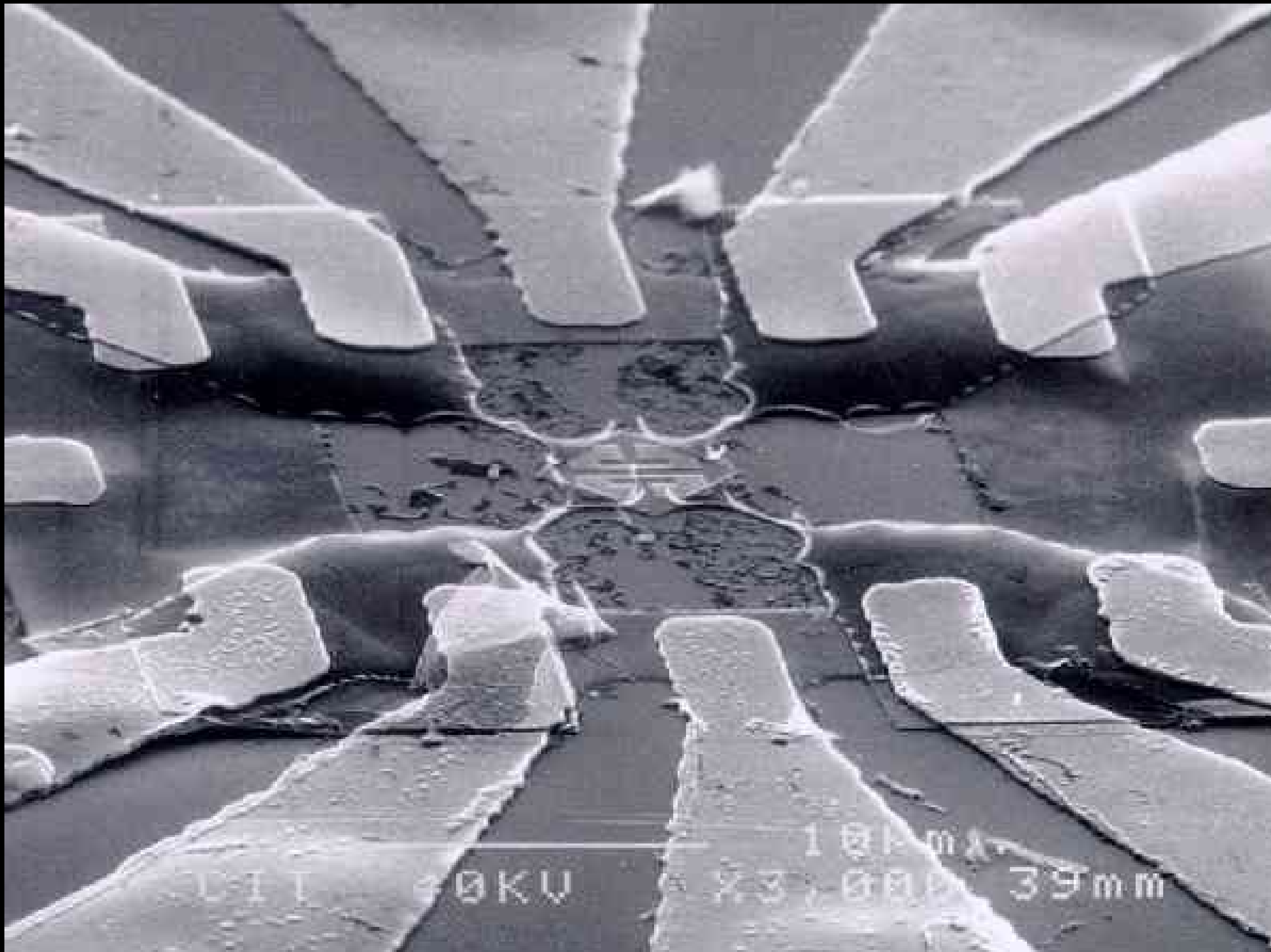
Quantum information processing and entanglement with Josephson charge qubits coupled through nanomechanical resonator

XuBo Zou, W. Mathis

Electromagnetic Theory Group at THT, Department of Electrical Engineering, University of Hannover, Germany

Received 5 January 2004; received in revised form 3 February 2004; accepted 5 February 2004
Communicated by R. Wu

Reality Check: A typical fabrication result!



What can we learn? Why do this?

•This teaches us how to engineer quantum limited detection where $h\omega$ is growing smaller and smaller (attempt at MHz) on systems that have huge numbers of degrees of freedom that must be controlled.

•This forces us to consider carefully the interaction between the measuring device and the measured quantum system. These studies will teach us intelligent measurement strategies (QND, indirect measurement, stroboscopic…..) (Quantum Engineering)

•Reveals the physics of decoherence and entanglements, relevant to the engineering of quantum coherent solid state devices (Quantum Computers?)

•This work will push the boundary between the classical world that we live in and the bizarre behavior that underlies reality (Foundations of Physics).

Will Quantum Mechanics break-down on large length scales?

My Group and Collaborators

Laboratory for Physical Sciences (LPS):

Marc Manheimer	NSA	Cooper-Pair Box
Carlos Sanchez	UMCP-grad student	
Akshay Niak	UMCP-grad student	
Ben Palmer	NSA	
Elinor Irish	Rochester-grad student	
Olivier Buu	post doc	
Matt LaHaye	UMCP-grad student	Nanomechanics
Patrick Truitt	UMCP-grad student	
Benedetta Camarota	post doc	
Alex Hutchinson	post doc	
Harish Bhaskaran	UMCP-grad student	Atomic Traps
Dan Stick	Univ. Michagan-grad	

Collaborators…

Michael Roukes and his group	Caltech
Chris Hammel and Denis Pelekhov	Ohio State
Miles Blencowe	Dartmouth
Andrew Armour	London College
Asa Hopkins, Kurt Jacobs, and Salman Habib	LANL
Ivar Martin	LANL
Halina Rubinsztein-Dunlop	Univ. of Queensland
Chris Monroe	Univ. of Michigan
Kamil Ekinci	Boston University
Pierre Echternach	JPL / Caltech

Outlook

- Entanglement with solid state qubits looks possible: mechanical superpositions
- Nanomechanical "QED" experiments look promising
- Are mechanical resonators useful as a quantum bus for charge qubits?
- Can we expand the domain of quantum mechanics?

This work is supported by National Security Agency (NSA)

Our Motto:

*Putting the **Spook** in the "Spooky actions at a distance"*

schwab@lps.umd.edu